Simple Stresses

Simple stresses are expressed as the ratio of the applied force divided by the resisting area or

$$\sigma = \text{Force} / \text{Area}.$$

It is the expression of force per unit area to structural members that are subjected to external forces and/or induced forces. Stress is the lead to accurately describe and predict the elastic deformation of a body.

Simple stress can be classified as normal stress, shear stress, and bearing stress. **Normal stress** develops when a force is applied perpendicular to the cross-sectional area of the material. If the force is going to pull the material, the stress is said to be **tensile stress** and **compressive stress** develops when the material is being compressed by two opposing forces. **Shear stress** is developed if the applied force is parallel to the resisting area. Example is the bolt that holds the tension rod in its anchor. Another condition of shearing is when we twist a bar along its longitudinal axis. This type of shearing is called torsion and covered in Chapter 3. Another type of simple stress is the **bearing stress**, it is the contact pressure between two bodies.

Suspension bridges are good example of structures that carry these stresses. The weight of the vehicle is carried by the bridge deck and passes the force to the stringers (vertical cables), which in turn, supported by the main suspension cables. The suspension cables then transferred the force into bridge towers.

Normal Stress

Stress

Stress is the expression of force applied to a unit area of surface. It is measured in psi (English unit) or in MPa (SI unit). Another unit of stress which is not commonly used is the dynes (cgs unit). Stress is the ratio of force over area.

stress = force / area

Simple Stresses

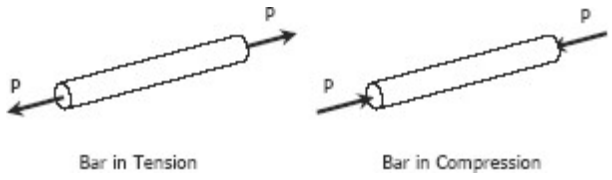
There are three types of simple stress namely; normal stress, shearing stress, and bearing stress.

Normal Stress

The resisting area is perpendicular to the applied force, thus normal. There are two types of normal stresses; tensile stress and compressive stress. Tensile stress applied to bar tends the bar to elongate while compressive stress tend to shorten the bar.

$$\sigma = \frac{P}{A}$$

where P is the applied normal load in Newton and A is the area in mm^2. The maximum stress in tension or compression occurs over a section normal to the load.



SOLVED PROBLEMS IN NORMAL STRESS

Problem 104

A hollow steel tube with an inside diameter of 100 mm must carry a tensile load of 400 kN. Determine the outside diameter of the tube if the stress is limited to 120 MN/m^2.

Solution 104

$P = \sigma A$

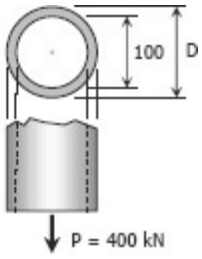
where:

$P = 400 \text{ kN} = 400\,000 \text{ N}$

$\sigma = 120 \text{ MPa}$

$A = \frac{1}{4}\pi D^2 - \frac{1}{4}\pi(100^2)$

$= \frac{1}{4}\pi(D^2 - 10\,000)$



thus,

$400\,000 = 120[\frac{1}{4}\pi(D^2 - 10\,000)]$

$400\,000 = 30\pi D^2 - 300\,000\pi$

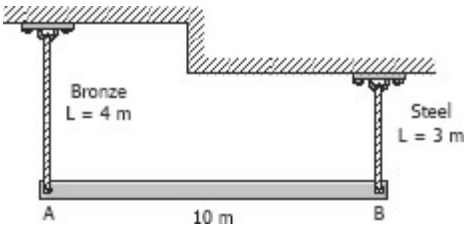
$D^2 = \dfrac{400\,000 + 300\,000\pi}{30\pi}$

$D = \mathbf{119.35 \text{ mm}}$

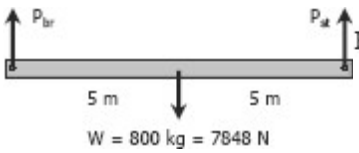
Problem 105

A homogeneous 800 kg bar AB is supported at either end by a cable as shown in Fig. P-105. Calculate the smallest area of each cable if the stress is not to exceed 90 MPa in bronze and 120 MPa in steel.

Figure P-105



Solution 105



By symmetry:

$P_{br} = P_{st} = \frac{1}{2}(7848)$

$= 3924 \text{ N}$

For bronze cable:

$P_{br} = \sigma_{br} A_{br}$

$3924 = 90 A_{br}$

$A_{br} = \mathbf{43.6 \text{ mm}^2}$

For steel cable:

$P_{st} = \sigma_{st} A_{st}$

$3924 = 120 A_{st}$

$A_{st} = \mathbf{32.7 \text{ mm}^2}$

Problem 106

The homogeneous bar shown in Fig. P-106 is supported by a smooth pin at C and a cable that runs from A to B around the smooth peg at D. Find the stress in the cable if its diameter is 0.6 inch and the bar weighs 6000 lb.

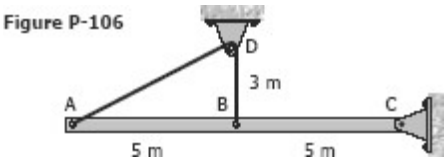
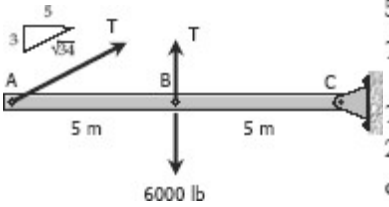


Figure P-106

Solution 106



$\Sigma M_C = 0$

$5T + 10(\frac{3}{\sqrt{34}}T) = 5(6000)$

$T = 2957.13$ lb

$T = \sigma A$

$2957.13 = \sigma[\frac{1}{4}\pi(0.6^2)]$

$\sigma = \mathbf{10\,458.72\ psi}$

Problem 107

A rod is composed of an aluminum section rigidly attached between steel and bronze sections, as shown in Fig. P-107. Axial loads are applied at the positions indicated. If P = 3000 lb and the cross sectional area of the rod is 0.5 in^2, determine the stress in each section.

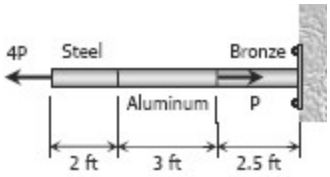
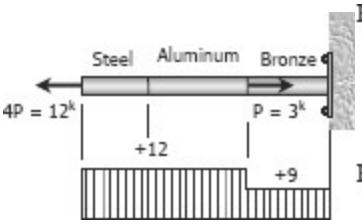


Figure P-107

Solution 107



For steel:

$\sigma_{st} A_{st} = P_{st}$

$\sigma_{st}(0.5) = 12$

$\sigma_{st} = \mathbf{24\ ksi}$

For aluminum:

$\sigma_{al} A_{al} = P_{al}$

$\sigma_{al}(0.5) = 12$

$\sigma_{al} = \mathbf{24\ ksi}$

For bronze:

$\sigma_{br} A_{br} = P_{br}$

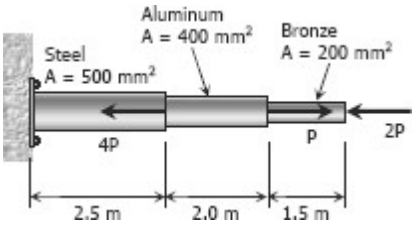
$\sigma_{br}(0.5) = 9$

$\sigma_{br} = \mathbf{18\ ksi}$

Problem 108

An aluminum rod is rigidly attached between a steel rod and a bronze rod as shown in Fig. P-108. Axial loads are applied at the positions indicated. Find the maximum value of P that will not exceed a stress in steel of 140 MPa, in aluminum of 90 MPa, or in bronze of 100 MPa.

Figure P-108



Solution 108

For bronze:

$\sigma_{br} A_{br} = 2P$

$100(200) = 2P$

$P = 10\,000$ N

For aluminum:

$\sigma_{al} A_{al} = P$

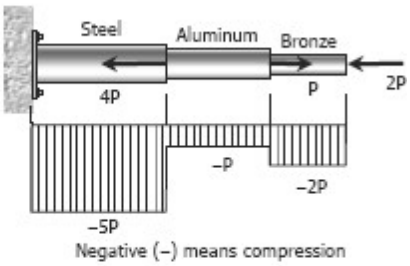
$90(400) = P$

$P = 36\,000$ N

For Steel:

$\sigma_{st} A_{st} = 5P$

$P = 14\,000$ N



For safe P, use $P = 10\,000$ N = **10 kN**

Problem 109

Determine the largest weight W that can be supported by two wires shown in Fig. P-109. The stress in either wire is not to exceed 30 ksi. The cross-sectional areas of wires AB and AC are 0.4 in^2 and 0.5 in^2, respectively.

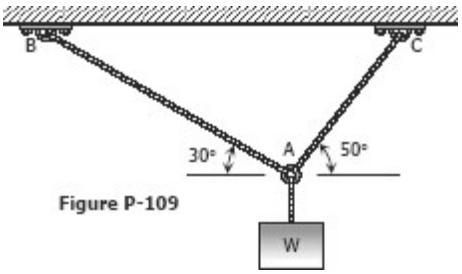


Figure P-109

Solution 109

For wire AB:

By sine law (from the force polygon):

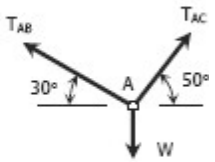
$$\frac{T_{AB}}{\sin 40°} = \frac{W}{\sin 80°}$$

$T_{AB} = 0.6527W$

$\sigma_{AB}A_{AB} = 0.6527W$

$30(0.4) = 0.6527W$

$W = 18.4$ kips



FBD of knot A

For wire AC:

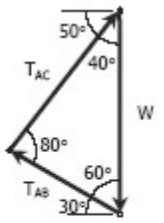
$$\frac{T_{AC}}{\sin 60°} = \frac{W}{\sin 80°}$$

$T_{AC} = 0.8794W$

$T_{AC} = \sigma_{AC}A_{AC}$

$0.8794W = 30(0.5)$

$W = 17.1$ kips



Force polygon of forces on knot A

Safe load $W = 17.1$ kips

Problem 110

A 12-inches square steel bearing plate lies between an 8-inches diameter wooden post and a concrete footing as shown in Fig. P-110. Determine the maximum value of the load P if the stress in wood is limited to 1800 psi and that in concrete to 650 psi.

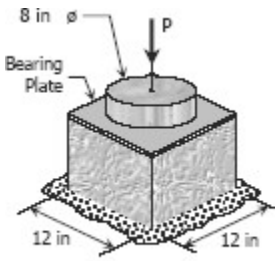


Figure P-110

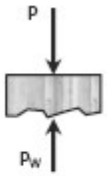
Solution 110

For wood:

$P_w = \sigma_w A_w$

$= 1800[\frac{1}{4}\pi(8^2)]$

$= 90\,477.9$ lb



FBD of Wood

From *FBD* of Wood:

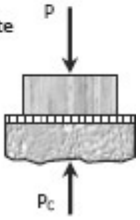
$P = P_w = 90\,477.9$ lb

For concrete:

$P_c = \sigma_c A_c$

$= 650(12^2)$

$= 93\,600$ lb



FBD of Concrete

From FBD of Concrete:

$P = P_c = 93\,600$ lb

Safe load $P = 90\,478$ lb

Problem 111

For the truss shown in Fig. P-111, calculate the stresses in members CE, DE, and DF. The crosssectional area of each member is 1.8 in^2. Indicate tension (T) or compression (C).

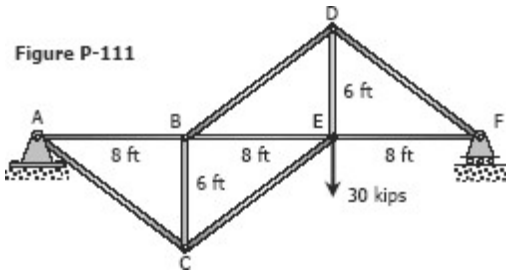
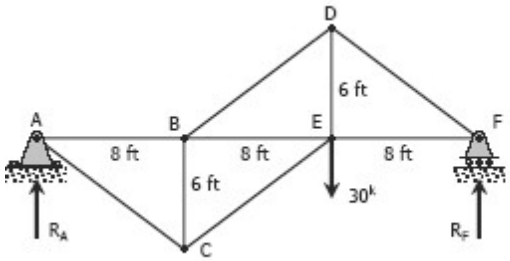


Figure P-111

Solution 111



Free Body Diagram of Truss

From the *FBD* of the truss:

$\Sigma M_A = 0$

$24R_B = 16(30)$

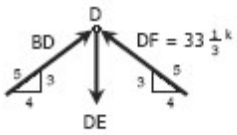
$R_F = 20^k$

At joint *F*:

$\Sigma F_V = 0$

$\frac{3}{5} DF = 20$

$DF = 33\frac{1}{3}{}^k\ (C)$



Joint D

At joint *D*: (by symmetry)

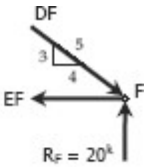
$BD = DF = 33\frac{1}{3}{}^k\ (C)$

$\Sigma F_V = 0$

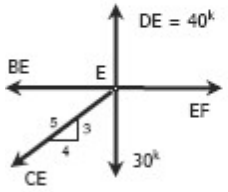
$DE = \frac{3}{5} BD + \frac{3}{5} DF$

$= \frac{3}{5}(33\frac{1}{3}) + \frac{3}{5}(33\frac{1}{3})$

$= 40^k\ (T)$



Joint F



Joint E

At joint *E*:

$\Sigma F_V = 0$

$\frac{3}{5} CE + 30 = 40$

$CE = 16\frac{2}{3}{}^k\ (T)$

Stresses:

Stress = Force/Area

$$\sigma_{CE} = \frac{16\frac{2}{3}}{1.8} = 9.26 \text{ ksi } (T)$$

$$\sigma_{DE} = \frac{40}{1.8} = 22.22 \text{ ksi } (T)$$

$$\sigma_{DF} = \frac{33\frac{1}{3}}{1.8} = 18.52 \text{ ksi } (C)$$

Problem 112

Determine the crosssectional areas of members AG, BC, and CE for the truss shown in Fig. P-112 above. The stresses are not to exceed 20 ksi in tension and 14 ksi in compression. A reduced stress in compression is specified to reduce the danger of buckling.

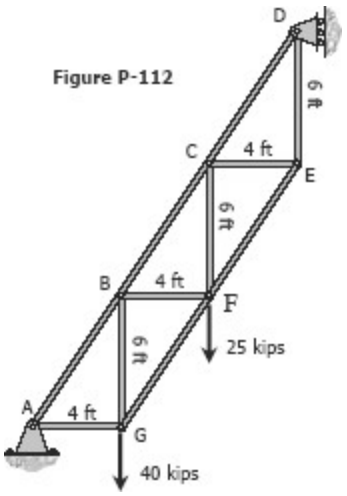
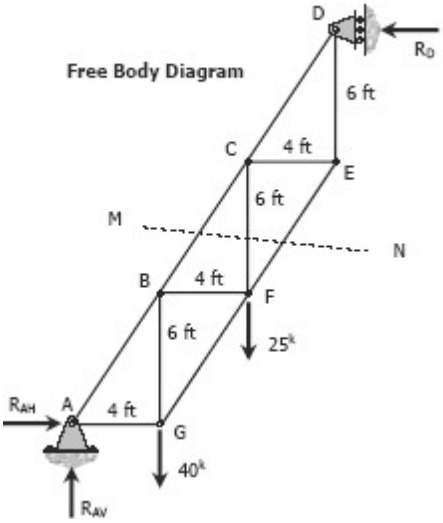


Figure P-112

Solution 112

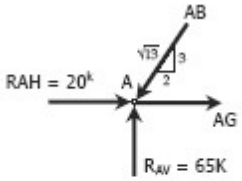


$\Sigma F_V = 0$
$R_{AV} = 40 + 25$
$= 65^k$

$\Sigma M_A = 0$
$18R_D = 8(25) + 4(40)$
$R_D = 20^k$

$\Sigma F_H = 0$
$R_{AH} = R_D = 20^k$

Check:
$\Sigma M_D = 0$
$12R_{AV} = 18(R_{AH}) + 4(25) + 8(40)$
$12(65) = 18(20) + 4(25) + 8(40)$
780 ft·kip = 780 ft·kip (OK!)



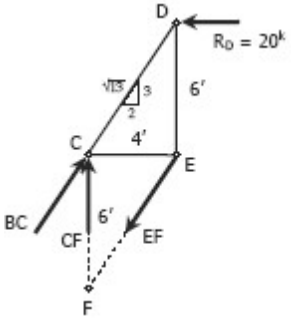
Joint A

For member AG:
At joint A:
$\Sigma F_V = 0$
$\frac{3}{\sqrt{13}} AB = 65$

$AB = \frac{65\sqrt{13}}{3}$
$= 78.12^k$

$\Sigma F_H = 0$
$AG + 20 = \frac{2}{\sqrt{13}} AB$
$AG = \frac{2}{\sqrt{13}} (78.12) - 20$
$= 20.33^k$ Tension

$AG = \sigma_{tension} A_{AG}$
$20.33 = 20\, A_{AG}$
$A_{AG} = \mathbf{1.17\ in^2}$

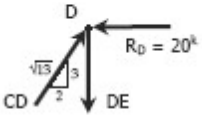


Section through MN

For member BC:
At section through MN
$\Sigma M_F = 0$
$6(\frac{2}{\sqrt{13}} BC) = 12(20)$
$BC = 20\sqrt{13}$
$= 72.11^k$ Compression

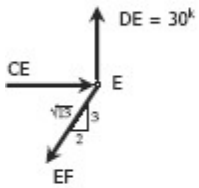
$BC = \sigma_{compression} A_{BC}$
$72.11 = 14\, A_{BC}$
$A_{BC} = \mathbf{5.15\ in^2}$

For member CE:
At joint D:
$\Sigma F_H = 0$
$\frac{2}{\sqrt{13}} CD = 20$
$CD = 10\sqrt{13}$
$= 36.06^k$



Joint D

$\Sigma F_V = 0$
$DE = \frac{3}{\sqrt{13}} CD$
$= \frac{3}{\sqrt{13}} (36.06)$
$= 30^k$



Joint E

At joint E:
$\Sigma F_V = 0$
$\frac{3}{\sqrt{13}} EF = 30$
$EF = 10\sqrt{13} = 36.06^k$

$\Sigma F_H = 0$
$CE = \frac{2}{\sqrt{13}} EF$
$= \frac{2}{\sqrt{13}} (36.06)$
$= 20^k$ Compression

$CF = \sigma_{compression} A_{CE}$
$20 = 14\, A_{CE}$
$A_{CE} = \mathbf{1.43\ in^2}$

Problem 113

Find the stresses in members BC, BD, and CF for the truss shown in Fig. P-113. Indicate the tension or compression. The cross sectional area of each member is 1600 mm^2.

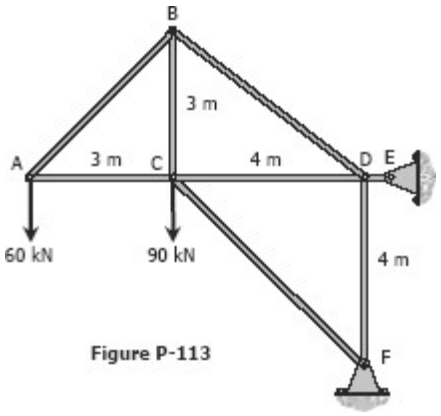
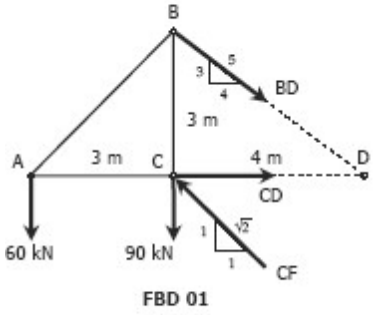


Figure P-113

Solution 113



FBD 01

For member BD: (See FBD 01)

$\Sigma M_C = 0$

$3(\frac{4}{5} BD) = 3(60)$

$BD = 75$ kN Tension

$BD = \sigma_{BD} A$

$75\ (1000) = \sigma_{BD}\ (1600)$

$\sigma_{BD} = 46.875$ MPa (Tension)

For member CF: (See FBD 01)

$\Sigma M_D = 0$

$4(\frac{1}{\sqrt{2}} CF) = 4(90) + 7(60)$

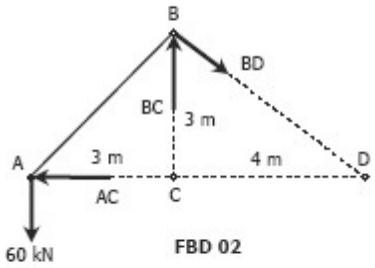
$CF = 195\sqrt{2}$

$= 275.77$ kN Compression

$CF = \sigma_{CF} A$

$275.77\ (1000) = \sigma_{CF}\ (1600)$

$\sigma_{CF} = 172.357$ MPa (Compression)



FBD 02

For member BC: (See FBD 02)

$\Sigma M_D = 0$

$4BC = 7(60)$

$BC = 105$ kN Compression

$BC = \sigma_{BC} A$

$105\ (1000) = \sigma_{BC}\ (1600)$

$\sigma_{BC} = 65.625$ MPa (Compression)

Problem 114

The homogeneous bar ABCD shown in Fig. P-114 is supported by a cable that runs from A to B around the smooth peg at E, a vertical cable at C, and a smooth inclined surface at D. Determine the mass of the heaviest bar that can be supported if the stress in each cable is limited to 100 MPa. The area of the cable AB is 250 mm^2 and that of the cable at C is 300 mm^2.

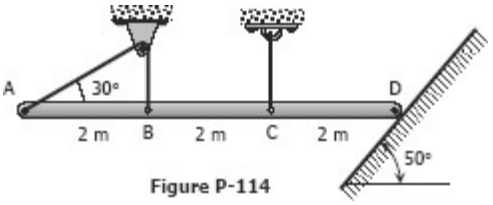
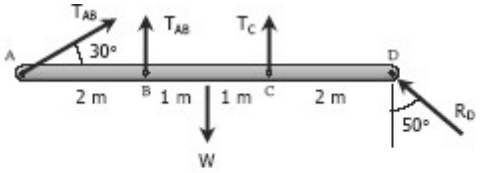


Figure P-114

Solution 114



$\Sigma F_H = 0$
$T_{AB} \cos 30° = R_D \sin 50°$
$R_D = 1.1305 T_{AB}$

$\Sigma F_V = 0$
$T_{AB} \sin 30° + T_{AB} + T_C + R_D \cos 50° = W$
$T_{AB} \sin 30° + T_{AB} + T_C + (1.1305 T_{AB}) \cos 50° = W$
$2.2267 T_{AB} + T_C = W$
$T_C = W - 2.2267 T_{AB}$

$\Sigma M_D = 0$
$6(T_{AB} \sin 30°) + 4T_{AB} + 2T_C = 3W$
$7T_{AB} + 2(W - 2.2267 T_{AB}) = 3W$
$2.5466 T_{AB} = W$
$T_{AB} = 0.3927W$

$T_C = W - 2.2267 T_{AB}$
$= W - 2.2267(0.3927W)$
$= 0.1256W$

Based on cable AB:
$T_{AB} = \sigma_{AB} A_{AB}$
$0.3927W = 100(250)$
$W = 63\,661.83$ N

Based on cable at C:
$T_2 = \sigma_C A_C$
$0.1256W = 100(300)$
$W = 238\,853.50$ N

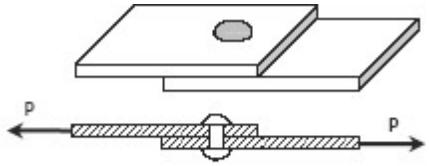
Safe weight $W = 63\,669.92$ N
$W = mg$
$63\,669.92 = m\,(9.81)$
$m = 6\,490$ kg
$= \mathbf{6.49\ Mg}$

Shearing Stress

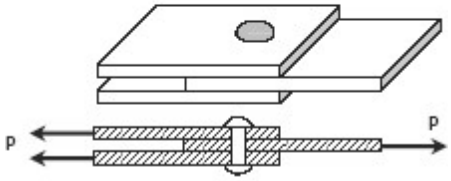
Forces parallel to the area resisting the force cause shearing stress. It differs to tensile and compressive stresses, which are caused by forces perpendicular to the area on which they act. Shearing stress is also known as tangential stress.

$$\tau = \frac{V}{A}$$

where V is the resultant shearing force which passes which passes through the centroid of the area A being sheared.



Single Shear



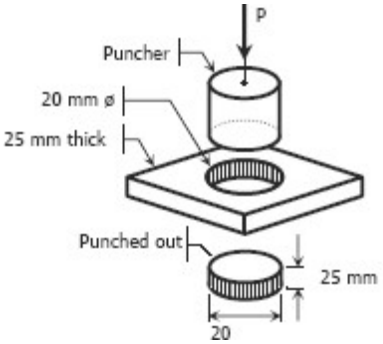
Double Shear

SOLVED PROBLEMS IN SHEARING STRESS

Problem 115

What force is required to punch a 20-mm-diameter hole in a plate that is 25 mm thick? The shear strength is 350 MN/m^2.

Solution 115



The resisting area is the shaded area along the perimeter and the shear force V is equal to the punching force P.

$$V = \tau A$$
$$P = 350[\pi(20)(25)]$$
$$= 549\,778.7 \text{ N}$$
$$= \mathbf{549.8 \text{ kN}}$$

Problem 116

As in Fig. 1-11c, a hole is to be punched out of a plate having a shearing strength of 40 ksi. The compressive stress in the punch is limited to 50 ksi. (**a**) Compute the maximum thickness of plate in which a hole 2.5 inches in diameter can be punched. (**b**) If the plate is 0.25 inch thick, determine the diameter of the smallest hole that can be punched.

Solution 116

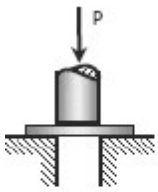


Figure 1-11c

(a) Maximum thickness of plate:
Based on puncher strength:

$P = \sigma A$
$= 50[\frac{1}{4}\pi(2.5^2)]$
$= 78.125\pi$ kips → Equivalent shear force of the plate

Based on shear strength of plate:

$V = \tau A$ → V = P
$78.125\pi = 40[\pi(2.5t)]$
$t = 0.781$ inch

(b) Diameter of smallest hole:
Based on compression of puncher:

$P = \sigma A$
$= 50(\frac{1}{4}\pi d^2)$
$= 12.5\pi d^2$ → Equivalent shear force for plate

Based on shearing of plate:

$V = \tau A$ → V = P
$12.5\pi d^2 = 40[\pi d(0.25)]$
$d = 0.8$ in

Problem 117

Find the smallest diameter bolt that can be used in the clevis shown in Fig. 1-11b if P = 400 kN. The shearing strength of the bolt is 300 MPa.

Solution 117

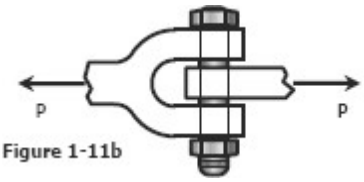


Figure 1-11b

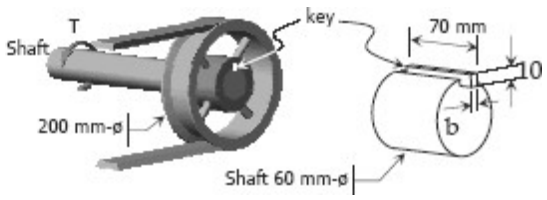
The bolt is subject to double shear.

$V = \tau A$
$400(1000) = 300[2(\frac{1}{4}\pi d^2)]$
$d = 29.13$ mm

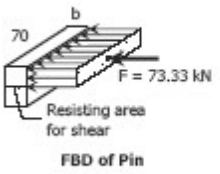
Problem 118

A 200-mm-diameter pulley is prevented from rotating relative to 60-mm-diameter shaft by a 70-mm-long key, as shown in Fig. P-118. If a torque T = 2.2 kN·m is applied to the shaft, determine the width b if the
allowable shearing stress in the key is 60 MPa.

Figure P-118



Solution 118

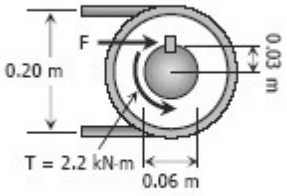


FBD of Pin

$T = 0.03F$
$2.2 = 0.03F$
$F = 73.33$ kN

$V = \tau A$
Where: V = F = 73.33 kN
A = 70b; τ = 60 MPa

$73.33(1000) = 60(70b)$
$b = \mathbf{17.46\ mm}$



Problem 119

Compute the shearing stress in the pin at B for the member supported as shown in Fig. P-119. The pin diameter is 20 mm.

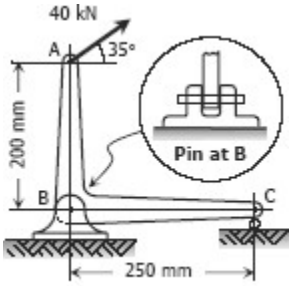
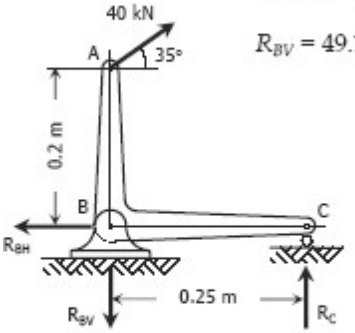


Figure P-119

Solution 119

From the FBD:
$\Sigma M_C = 0$
$0.25R_{BV} = 0.25(40 \sin 35°) + 0.2(40 \cos 35°)$
$R_{BV} = 49.156$ kN



Free Body Diagram

$\Sigma F_H = 0$
$R_{BH} = 40 \cos 35°$
$= 32.766$ kN

$R_B = \sqrt{R_{BH}^2 + R_{BV}^2}$
$= \sqrt{32.766^2 + 49.156^2}$
$= 59.076$ kN → shear force of pin at B

$V_B = \tau_B A$ → double shear
$59.076\ (1000) = \tau_B \{2[\frac{1}{4}\pi(20^2)]\}$
$\tau_B = \mathbf{94.02\ MPa}$

Problem 120

The members of the structure in Fig. P-120 weigh 200 lb/ft. Determine the smallest diameter pin that can be used at A if the shearing stress is limited to 5000 psi. Assume single shear.

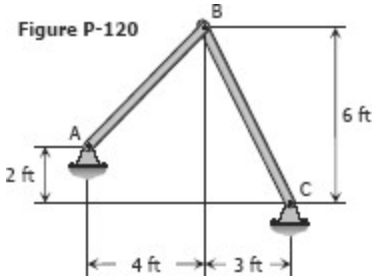


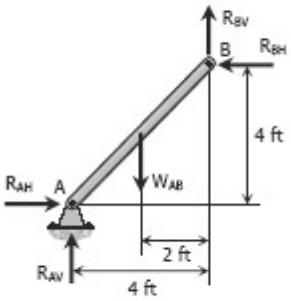
Figure P-120

Solution 120

For member AB:

Length, $L_{AB} = \sqrt{4^2 + 4^2}$
$= 5.66$ ft

Weight, $W_{AB} = 5.66(200)$
$= 1132$ lb



FBD of member

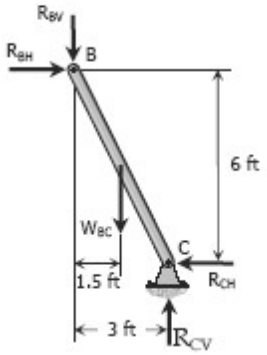
$\Sigma M_A = 0$
$4R_{BH} + 4R_{BV} = 2W_{AB}$
$4R_{BH} + 4R_{BV} = 2(1132)$
$R_{BH} + R_{BV} = 566$ → (1)

For member BC:

Length, $L_{BC} = \sqrt{3^2 + 6^2}$
$= 6.71$ ft

Weight, $W_{BC} = 6.71(200)$
$= 1342$ lb

$\Sigma M_C = 0$
$6R_{BH} = 1.5W_{BC} + 3R_{BV}$
$6R_{BH} - 3R_{BV} = 1.5(1342)$
$2R_{BH} - R_{BV} = 671$ → (2)



FBD of member BC

Add equations (1) and (2)

$R_{BH} + R_{BV} = 566$ → (1)
$2R_{BH} - R_{BV} = 671$ → (2)
$3R_{BH} = 1237$
$R_{BH} = 412.33$ lb

From equation (1):
$412.33 + R_{BV} = 566$
$R_{BV} = 153.67$ lb

From the FBD of member AB
$\Sigma F_H = 0$
$R_{AH} = R_{BH} = 412.33$ lb

$\Sigma F_V = 0$
$R_{AV} + R_{BV} = W_{AB}$
$R_{AV} + 153.67 = 1132$
$R_{AV} = 978.33$ lb

$$R_A = \sqrt{R_{AH}^2 + R_{AV}^2}$$
$$= \sqrt{412.33^2 + 978.33^2}$$
$$= 1061.67 \text{ lb} \quad \rightarrow \text{shear force of pin at A}$$

$$V = \tau A$$
$$1061.67 = 5000(\tfrac{1}{4}\pi d^2)$$
$$d = \mathbf{0.520 \text{ in}}$$

Problem 121

Referring to Fig. P-121, compute the maximum force P that can be applied by the machine operator, if the shearing stress in the pin at B and the axial stress in the control rod at C are limited to 4000 psi and 5000 psi, respectively. The diameters are 0.25 inch for the pin, and 0.5 inch for the control rod. Assume single shear for the pin at B.

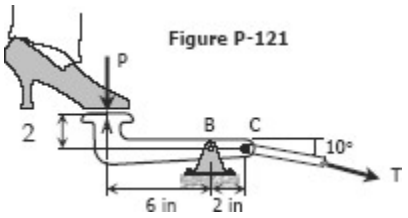


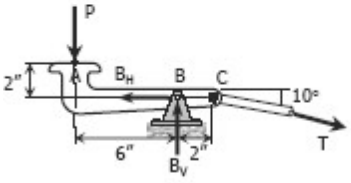
Figure P-121

Solution 121

$$[\Sigma M_B = 0] \quad 6P = 2T \sin 10° \quad \rightarrow (1)$$

$$[\Sigma F_H = 0] \quad B_H = T \cos 10° \quad \rightarrow \text{from (1), T = 3P/sin 10°}$$
$$B_H = (3P/\sin 10°) \cos 10°$$
$$B_H = 3 \cot 10° \, P$$

$$[\Sigma F_V = 0] \quad B_V = T \sin 10° + P \rightarrow \text{from (1), T sin 10° = 3P}$$
$$B_V = 3P + P$$
$$B_V = 4P$$



$$R_B^2 = B_H^2 + B_V^2$$
$$R_B^2 = (3 \cot 10° \, P)^2 + (4P)^2$$
$$R_B^2 = 305.47P^2$$
$$R_B = 17.48P$$
$$P = R_B/17.48 \quad \rightarrow (2)$$

Based on tension of rod (equation 1):

$$P = \tfrac{1}{3} T \sin 10°$$
$$P = \tfrac{1}{3}[5000 \times \tfrac{1}{4}\pi(0.5)^2] \sin 10°$$
$$P = 56.83 \text{ lb}$$

Based on shear of rivet (equation 2):

$$P = 4000 \times \tfrac{1}{4}\pi(0.25)^2 / 17.48$$
$$P = 11.23 \text{ lb}$$

Safe load $P = \mathbf{11.23 \text{ lb}}$

Problem 122

Two blocks of wood, width w and thickness t, are glued together along the joint inclined at the angle θ as shown in Fig. P-122. Using the free-body diagram concept in Fig. 1-4a, show that the shearing stress on the glued joint is $\tau = P \sin 2\theta / 2A$, where A is the cross-sectional area.

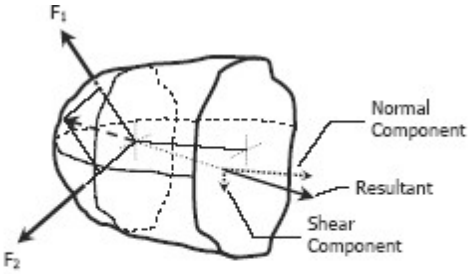


Figure 1-4a Normal and shear components of the resultant on arbitrary section.

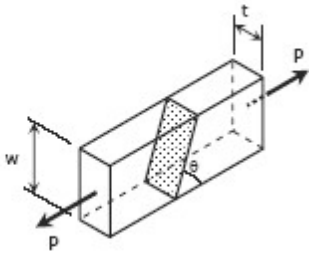
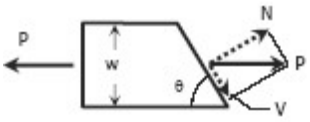


Figure P-122

Solution 122



Shear area, $A_{shear} = t\,(w \csc \theta)$
$= tw \csc \theta$
$= A \csc \theta$

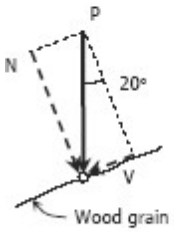
Shear force, $V = P \cos \theta$

$V = \tau A_{shear}$
$P \cos \theta = \tau (A \csc \theta)$
$\tau = P \sin \theta \cos \theta / A$
$= P\,(2 \sin \theta \cos \theta) / 2A$
$= P \sin 2\theta / 2A$ *(ok!)*

Problem 123

A rectangular piece of wood, 50 mm by 100 mm in cross section, is used as a compression block shown in Fig. P-123. Determine the axial force P that can be safely applied to the block if the compressive stress in wood is limited to 20 MN/m^2 and the shearing stress parallel to the grain is limited to 5 MN/m^2. The grain makes an angle of 20° with the horizontal, as shown. (Hint: Use the results in Problem 122.)

Solution 123



Based on maximum compressive stress:

Normal force:

$N = P \cos 20°$

Normal area:

$A_N = 50\ (100 \sec 20°)$

$= 5320.89\ \text{mm}^2$

$N = \sigma A_N$

$P \cos 20° = 20\ (5320.89)$

$P = 113\ 247\ \text{N}$

$= 133.25\ \text{kN}$

Based on maximum shearing stress:

Shear force:

$V = P \sin 20°$

Shear area:

$A_V = A_N$

$= 5320.89\ \text{mm}^2$

$V = \tau A_V$

$P \sin 20° = 5\ (5320.89)$

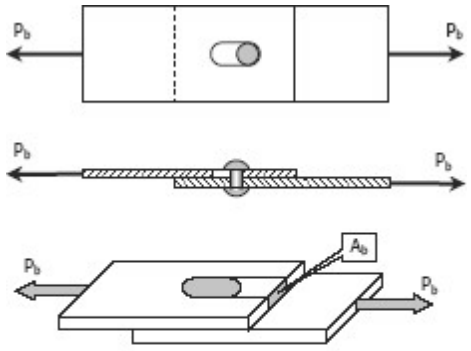
$P = 77\ 786\ \text{N}$

$= 77.79\ \text{kN}$

For safe compressive force, use $P = 77.79\ \text{kN}$

Bearing Stress

Bearing stress is the contact pressure between the separate bodies. It differs from compressive stress, as it is an internal stress caused by compressive forces.



$$\sigma_b = \frac{P_b}{A_b}$$

SOLVED PROBLEMS IN BEARING STRESS

Problem 125

In Fig. 1-12, assume that a 20-mm-diameter rivet joins the plates that are each 110 mm wide. The allowable stresses are 120 MPa for bearing in the plate material and 60 MPa for shearing of rivet. Determine (a) the minimum thickness of each plate; and (b) the largest average tensile stress in the plates.

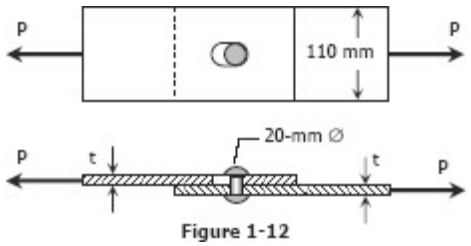


Figure 1-12

Solution 125

(a) From shearing of rivet:

$P = \tau A_{rivets}$

$= 60[\frac{1}{4}\pi(20^2)]$

$= 6000\pi$ N

From bearing of plate material:

$P = \sigma_b A_b$

$6000\pi = 120(20t)$

$t = 7.85$ mm

(b) Largest average tensile stress in the plate:

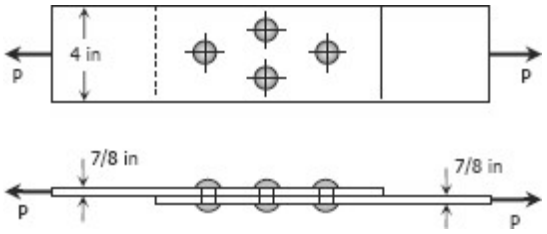
$P = \sigma A$

$6000\pi = \sigma[7.85(110 - 20)]$

$\sigma = 26.67$ MPa

Problem 126

The lap joint shown in Fig. P-126 is fastened by four ¾-in.-diameter rivets. Calculate the maximum safe load P that can be applied if the shearing stress in the rivets is limited to 14 ksi and the bearing stress in the plates is limited to 18 ksi. Assume the applied load is uniformly distributed among the four rivets.



Solution 126

Based on shearing of rivets:

$P = \tau A$

$P = 14[4\,(\frac{1}{4}\pi)(\frac{3}{4})^2\,]$

$P = 24.74$ kips

Based on bearing of plates:

$P = \sigma_b\, A_b$

$P = 18[4\,(\frac{3}{4})(\frac{7}{8})\,]$

$P = 47.25$ kips

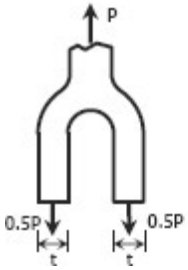
Safe load $P = \mathbf{24.74}$ **kips**

Problem 127

In the clevis shown in Fig. 1-11b, find the minimum bolt diameter and the minimum thickness of each yoke that will support a load P = 14 kips without exceeding a shearing stress of 12 ksi and a bearing stress of 20 ksi.

Figure 1-11b

Solution 127



For shearing of rivets (double shear)

$P = \tau A$

$14 = 12[2(\frac{1}{4}\pi d^2)]$

$d = 0.8618$ in → diameter of bolt

For bearing of yoke:

$P = \sigma_b A_b$

$14 = 20[2(0.8618t)]$

$t = 0.4061$ in → thickness of yoke

Problem 128

A W18 × 86 beam is riveted to a W24 × 117 girder by a connection similar to that in Fig. 1-13. The diameter of the rivets is 7/8 in., and the angles are each 4 × 31/2 × 3/8 in. For each rivet, assume that the allowable stresses are τ = 15 ksi and σ_b = 32 ksi. Find the allowable

load on the connection.

Figure 1-13

Solution 128

Note: Textbook is Strength of Materials 4th edition by Pytel and Singer

Relevant data from the table (Appendix *B* of textbook): *Properties of Wide-Flange Sections (W shapes): U.S. Customary Units*

Designation	Web thickness
W18 × 86	0.480 in
W24 × 117	0.550 in

Shearing strength of rivets:
There are 8 single-shear rivets in the girder and 4 double-shear (equivalent to 8 single-shear) in the beam, thus, the shear strength of rivets in girder and beam are equal.

$V = \tau A = 15[\frac{1}{4}\pi(\frac{7}{8})^2(8)]$

$V = 72.16$ kips

Bearing strength on the girder:
The thickness of girder W24 × 117 is 0.550″ while that of the angle clip L4 × $3\frac{1}{2}$ × $\frac{3}{8}$ is $\frac{3}{8}$″ or 0.375″, thus, the critical in bearing is the clip.

$P = \sigma_b A_b = 32[\frac{7}{8}(0.375)(8)]$

$P = 84$ kips

Bearing strength on the beam:
The thickness of beam W18 × 86 is 0.480″ while that of the clip angle is 2 × 0.375″ = 0.75″ (clip angles are on both sides of the beam), thus, the critical in bearing is the beam.

$P = \sigma_b A_b = 32[\frac{7}{8}(0.480)(4)]$

$P = 53.76$ kips

The allowable load on the connection is $P = 53.76$ kips

Problem 129

A 7/8-in.-diameter bolt, having a diameter at the root of the threads of 0.731 in., is used to fasten two timbers together as shown in Fig. P-129. The nut is tightened to cause a tensile stress of 18 ksi in the bolt. Compute the shearing stress in the head of the bolt and in the threads. Also, determine the outside diameter of the washers if their inside diameter is 9/8 in. and the bearing stress is limited to 800 psi.

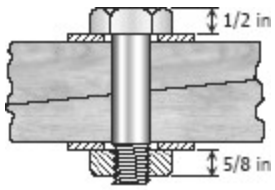
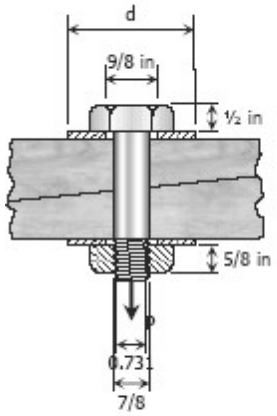


Figure P-127

Solution 129



Tensile force on the bolt:

$P = \sigma A = 18[\frac{1}{4}\pi(\frac{7}{8})^2]$

$P = 10.82$ kips

Shearing stress in the head of the bolt:

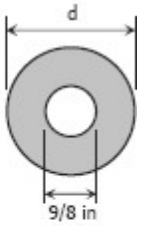
$$\tau = \frac{P}{A} = \frac{10.82}{\pi(\frac{7}{8})(\frac{1}{2})}$$

$\tau = 7.872$ ksi

Shearing stress in the threads:

$$\tau = \frac{P}{A} = \frac{10.82}{\pi(0.731)(\frac{5}{8})}$$

$\tau = 7.538$ ksi



Washer

Outside diameter of washer

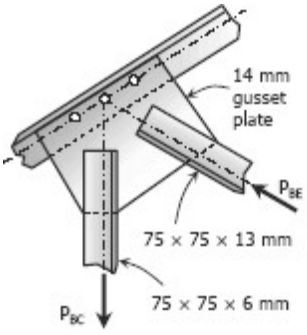
$P = \sigma_b A_b$

$10.82(1000) = 800\{\frac{1}{4}\pi[d^2 - (\frac{9}{8})^2]\}$

$d = 4.3$ in

Problem 130

Figure P-130 shows a roof truss and the detail of the riveted connection at joint B. Using allowable stresses of τ = 70 MPa and σ_b= 140 MPa, how many 19-mm diameter rivets are required to fasten member BC to the gusset plate? Member BE? What is the largest average tensile or compressive stress in BC and BE?



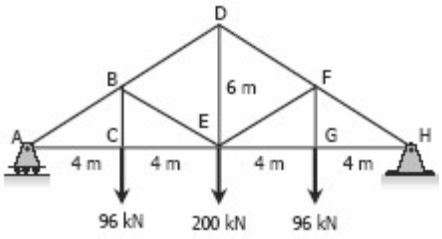
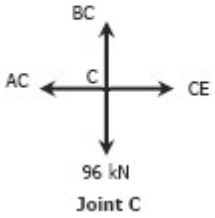


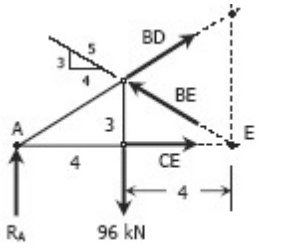
Figure P-130 and P-131

Solution 130



Joint C

At Joint C:
$\Sigma F_V = 0$
$BC = 96$ kN (Tension)



Section through BD, BE, and CE

Consider the section through member *BD*, *BE*, and *CE*:
$\Sigma M_A = 0$
$8(\frac{3}{5}BE) = 4(96)$
$BE = 80$ kN (Compression)

For Member *BC*:
Based on shearing of rivets:
$BC = \tau A$
Where A = area of 1 rivet × number of rivets, n
$96\,000 = 70[\frac{1}{4}\pi(19^2)n]$
$n = 4.8$ say 5 rivets

Based on bearing of member:
$BC = \sigma_b A_b$
Where A_b = diameter of rivet × thickness of BC × number of rivets, n
$96\,000 = 140[19(6)n]$
$n = 6.02$ say 7 rivets

use 7 rivets for member *BC*

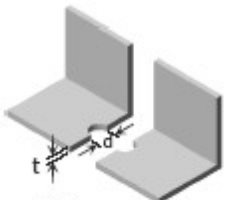
For member *BE*:
Based on shearing of rivets:
$BE = \tau A$
Where A = area of 1 rivet × number of rivets, n
$80\,000 = 70[\frac{1}{4}\pi(19^2)n]$
$n = 4.03$ say 5 rivets

Based on bearing of member:
$BE = \sigma_b A_b$
Where A_b = diameter of rivet × thickness of BE × number of rivets, n
$80\,000 = 140[19(13)n]$
$n = 2.3$ say 3 rivets

use 5 rivets for member *BE*

Relevant data from the table (Appendix B of textbook): *Properties of Equal Angle Sections: SI Units*

Designation	Area
L75 × 75 × 6	864 mm²
L75 × 75 × 13	1780 mm²



t = thickness of member
d = diameter of rivet hole

Note:
A = Area – dt

Tensile stress of member *BC* (L75 × 75 × 6):
$$\sigma = \frac{P}{A} = \frac{96(1000)}{864 - 19(6)}$$
$\sigma = 128$ Mpa

Compressive stress of member *BE* (L75 × 75 × 13):
$$\sigma = \frac{P}{A} = \frac{80(1000)}{1780}$$
$\sigma = 44.94$ Mpa

Problem 131

Repeat Problem 130 if the rivet diameter is 22 mm and all other data remain unchanged.

Solution 131

For member BC:

$P = 96$ kN (Tension)

Based on shearing of rivets:

$P = \tau A$

$96\,000 = 70[\frac{1}{4}\pi(22^2)n]$

$n = 3.6$ say 4 rivets

Based on bearing of member:

$P = \sigma_b A_b$

$96\,000 = 140[22(6)n]$

$n = 5.2$ say 6 rivets

Use 6 rivets for member BC

Tensile stress:

$$\sigma = \frac{P}{A} = \frac{96(1000)}{864 - 22(6)}$$

$\sigma = 131.15$ MPa

For member BE:

$P = 80$ kN (Compression)

Based on shearing of rivets:

$P = \tau A$

$80\,000 = 70[\frac{1}{4}\pi(22^2)n]$

$n = 3.01$ say 4 rivets

Based on bearing of member:

$P = \sigma_b A_b$

$80\,000 = 140[22(13)n]$

$n = 1.998$ say 2 rivets

use 4 rivets for member BE

Compressive stress:

$$\sigma = \frac{P}{A} = \frac{80(1000)}{1780}$$

$\sigma = 44.94$ MPa

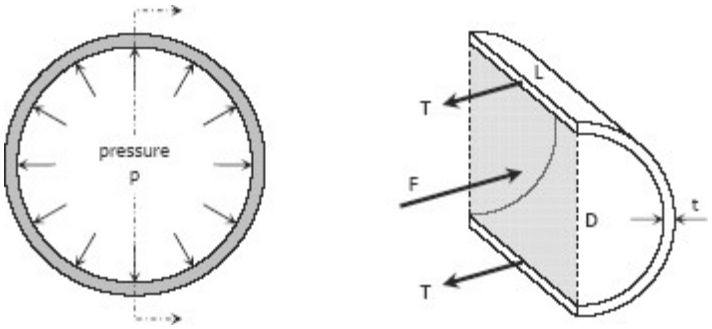
Thin-Walled Pressure Vessels

A tank or pipe carrying a fluid or gas under a pressure is subjected to tensile forces, which resist bursting, developed across longitudinal and transverse sections.

TANGENTIAL STRESS

(Circumferential Stress)

Consider the tank shown being subjected to an internal pressure p. The length of the tank is L and the wall thickness is t. Isolating the right half of the tank:



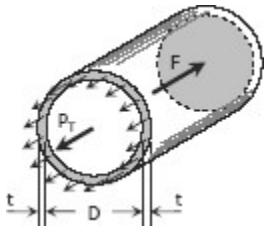
$$F = pA = pDL$$
$$T = \sigma_t A_{wall} = \sigma_t tL$$
$$[\Sigma F_H = 0]$$
$$F = 2T$$
$$pDL = 2(\sigma_t tL)$$
$$\sigma_t = \frac{pD}{2t}$$

If there exist an external pressure p_o and an internal pressure p_i, the formula may be expressed as:

$$\sigma_t = \frac{(p_i - p_o)D}{2t}$$

LONGITUDINAL STRESS, σ_L

Consider the free body diagram in the transverse section of the tank:



The total force acting at the rear of the tank F must equal to the total longitudinal stress on the wall $P_T = \sigma_L A_{wall}$. Since t is so small compared to D, the area of the wall is close to πDt

$$F = pA = p\frac{\pi}{4}D^2$$

$$P_T = \sigma_L \pi D t$$

$$[\Sigma F_H = 0]$$

$$P_T = F$$

$$\sigma_L \pi D t = p\frac{\pi}{4}D^2$$

$$\sigma_L = \frac{pD}{4t}$$

If there exist an external pressure p_o and an internal pressure p_i, the formula may be expressed as:

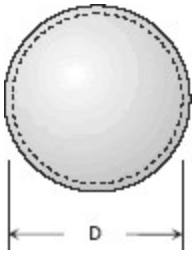
$$\sigma_L = \frac{(p_i - p_o)D}{4t}$$

It can be observed that the tangential stress is twice that of the longitudinal stress.

$$\sigma_t = 2\ \sigma_L$$

SPHERICAL SHELL

If a spherical tank of diameter D and thickness t contains gas under a pressure of p, the stress at the wall can be expressed as:

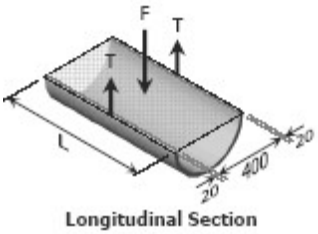


$$\sigma_L = \frac{(p_i - p_o)D}{4t}$$

Problem 133

A cylindrical steel pressure vessel 400 mm in diameter with a wall thickness of 20 mm, is subjected to an internal pressure of 4.5 MN/m^2. (a) Calculate the tangential and longitudinal stresses in the steel. (b) To what value may the internal pressure be increased if the stress in the steel is limited to 120 MN/m^2? (c) If the internal pressure were increased until the vessel burst, sketch the type of fracture that would occur.

Solution 133



Longitudinal Section

(a) Tangential stress (longitudinal section):

$$F = 2T$$

$$pDL = 2(\sigma_t\, tL)$$

$$\sigma_t = \frac{pD}{2t} = \frac{4.5(400)}{2(20)}$$

$$\sigma_t = \mathbf{45\ MPa}$$

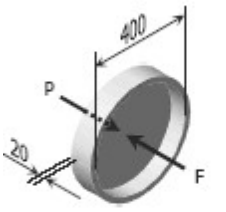
Longitudinal Stress (transverse section):

$$F = P$$

$$\tfrac{1}{4}\pi D^2 p = \sigma_l\,(\pi D t)$$

$$\sigma_l = \frac{pD}{4t} = \frac{4.5(400)}{4(20)}$$

$$\sigma_l = \mathbf{22.5\ MPa}$$



Transverse Section

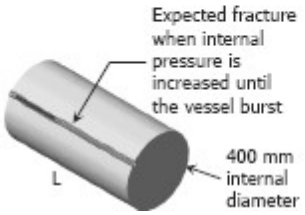
(b) From (a), $\sigma_t = \dfrac{pD}{2t}$ and $\sigma_l = \dfrac{pD}{4t}$ thus, $\sigma_t = 2\sigma_l$, this shows that tangential stress is the critical.

$$\sigma_t = \frac{pD}{2t}$$

$$120 = \frac{p(400)}{2(20)}$$

$$P = \mathbf{12\ MPa}$$

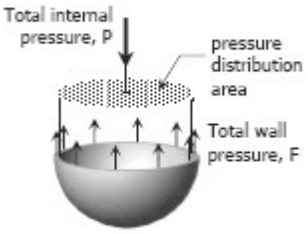
(c) The bursting force will cause a stress on the longitudinal section that is twice to that of the transverse section. Thus, fracture is expected as shown.



Problem 134

The wall thickness of a 4-ft-diameter spherical tank is 5/16 in. Calculate the allowable internal pressure if the stress is limited to 8000 psi.

Solution 134



Total internal pressure:

$$P = p\left(\tfrac{1}{4}\pi D^2\right)$$

Resisting wall:

$$F = P$$

$$\sigma A = p\left(\tfrac{1}{4}\pi D^2\right)$$

$$\sigma(\pi D t) = p\left(\tfrac{1}{4}\pi D^2\right)$$

$$\sigma = \frac{pD}{4t}$$

$$8000 = \frac{p(4\times 12)}{4\left(\frac{5}{16}\right)}$$

$$p = 208.33 \text{ psi}$$

Problem 135

Calculate the minimum wall thickness for a cylindrical vessel that is to carry a gas at a pressure of 1400 psi. The diameter of the vessel is 2 ft, and the stress is limited to 12 ksi.

Solution 135

The critical stress is the tangential stress

$$\sigma_t = \frac{pD}{2t}$$

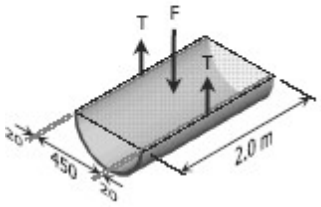
$$12\,000 = \frac{1400(2\times 12)}{2t}$$

$$t = 1.4 \text{ in}$$

Problem 136

A cylindrical pressure vessel is fabricated from steel plating that has a thickness of 20 mm. The diameter of the pressure vessel is 450 mm and its length is 2.0 m. Determine the maximum internal pressure that can be applied if the longitudinal stress is limited to 140 MPa, and the circumferential stress is limited to 60 MPa.

Solution 136



Based on circumferential stress (tangential):

$\Sigma F_V = 0$

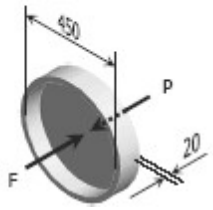
$F = 2T$

$p(DL) = 2(\sigma_t\, Lt)$

$$\sigma_t = \frac{pD}{2t}$$

$$60 = \frac{p(450)}{2(20)}$$

$p = 5.33$ MPa



Based on longitudinal stress:

$\Sigma F_H = 0$

$F = P$

$p\left(\frac{1}{4}\pi D^2\right) = \sigma_t(\pi Dt)$

$$\sigma_l = \frac{pD}{4t}$$

$$140 = \frac{p(450)}{4(20)}$$

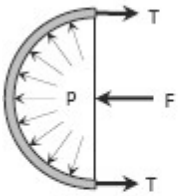
$p = 24.89$ MPa

Use $p = 5.33$ MPa

Problem 137

A water tank, 22 ft in diameter, is made from steel plates that are ½ in. thick. Find the maximum height to which the tank may be filled if the circumferential stress is limited to 6000 psi. The specific weight of water is 62.4 lb/ft^3.

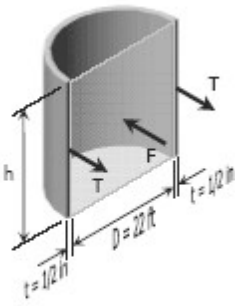
Solution 137



$\sigma_t = 6000$ psi

$$\sigma_t = \frac{6000\ \text{lb}}{\text{in}^2}\left(\frac{12\ \text{in}}{\text{ft}}\right)^2$$

$\sigma_t = 864\,000$ lb/ft^2



Assuming pressure distribution to be uniform:

$p = \gamma h = 62.4h$

$F = pA = 62.4h(Dh)$

$F = 62.4(22)h^2$

$F = 1372.8h^2$

$T = \sigma_t A_t = 864\,000(th)$

$T = 864\,000\,(\frac{1}{2}\times\frac{1}{12})\,h$

$T = 36\,000h$

$\Sigma F = 0$

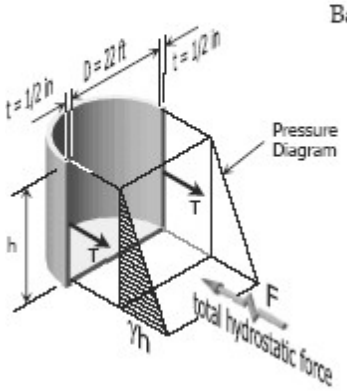
$F = 2T$

$1372.8h^2 = 2(36\,000h)$

$h = \mathbf{52.45}$ ft

Comment:

Given a free surface of water, the actual pressure distribution on the vessel is not uniform. It varies linearly from 0 at the free surface to γh at the bottom (see figure below). Using this actual pressure distribution, the total hydrostatic pressure is reduced by 50%. This reduction of force will take our design into critical situation; giving us a maximum height of 200% more than the h above.



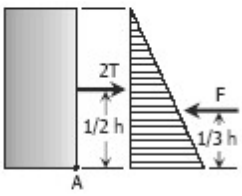
Based on actual pressure distribution:

Total hydrostatic force, F:

F = volume of pressure diagram

$F = \frac{1}{2}(\gamma h^2)D = \frac{1}{2}(62.4h^2)(22)$

$F = 686.4h^2$



$\Sigma M_A = 0$

$2T\,(\frac{1}{2}h) - F\,(\frac{1}{3}h) = 0$

$T = \frac{1}{3}F$

$\sigma_t\,(ht) = \frac{1}{3}(686.4h^2)$

$$h = \frac{3\sigma_t t}{686.4} = \frac{3(864\,000)(\frac{1}{2}\times\frac{1}{12})}{686.4}$$

$h = 157.34$ ft

Problem 138

The strength of longitudinal joint in Fig. 1-17 is 33 kips/ft, whereas for the girth is 16 kips/ft. Calculate the maximum diameter of the cylinder tank if the internal pressure is 150 psi.

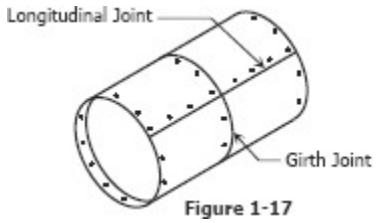


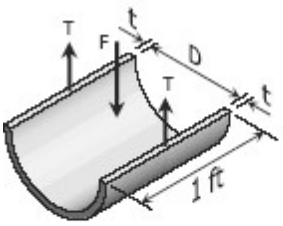
Figure 1-17

Solution 138

Internal pressure, p:

$$p = 150 \text{ psi} = \frac{150 \text{ lb}}{\text{in}^2}\left(\frac{12 \text{ in}}{\text{ft}}\right)^2$$

$$p = 21\,600 \text{ lb/ft}^2$$



For longitudinal joint (tangential stress):

Consider 1 ft length

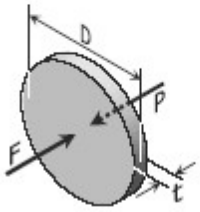
$$F = 2T$$

$$pD = 2\sigma_t t$$

$$\sigma_t = \frac{pD}{2t}$$

$$\frac{33000}{t} = \frac{21600\,D}{2t}$$

$$D = 3.06 \text{ ft} = 36.67 \text{ in}$$



For girth joint (longitudinal stress):

$$F = P$$

$$p\left(\tfrac{1}{4}\pi D^2\right) = \sigma_l\,(\pi D t)$$

$$\sigma_l = \frac{pD}{4t}$$

$$\frac{16000}{t} = \frac{21600\,D}{4t}$$

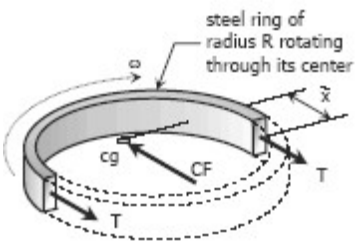
$$D = 2.96 \text{ ft} = 35.56 \text{ in.}$$

Use the smaller diameter, $D = 35.56$ in.

Problem 139

Find the limiting peripheral velocity of a rotating steel ring if the allowable stress is 20 ksi and steel weighs 490 lb/ft^3. At what revolutions per minute (rpm) will the stress reach 30 ksi if the mean radius is 10 in.?

Solution 139



FBD of Ring in Rotation

Centrifugal Force, CF:

$$CF = M\omega^2\bar{x}$$

where: $M = \frac{W}{g} = \frac{\gamma V}{g} = \frac{\gamma \pi RA}{g}$

$$\omega = v/R$$

$$\bar{x} = 2R/\pi$$

$$CF = \frac{\gamma \pi RA}{g}\left(\frac{v}{R}\right)^2\left(\frac{2R}{\pi}\right)$$

$$CF = \frac{2\gamma A v^2}{g}$$

$$2T = CF$$

$$2\sigma A = \frac{2\gamma A v^2}{g}$$

$$\sigma = \frac{\gamma v^2}{g}$$

From the given data:

σ = 20 ksi = (20 000 lb/in2)(12 in/ft)a^2

σ = 2 880 000 lb/ft2

γ = 490 lb/ft^3

$$2\,880\,000 = \frac{490 v^2}{32.2}$$

$$v = 435.04 \text{ ft/sec}$$

When σ = 30 ksi, and R = 10 in

$$\sigma = \frac{\gamma v^2}{g}$$

$$30\,000(12^2) = \frac{490 v^2}{32.2}$$

$$v = 532.81 \text{ ft/sec}$$

$$\omega = v/R = \frac{532.81}{10/12}$$

$$\omega = 639.37 \text{ rad/sec}$$

$$\omega = \frac{639.37 \text{ rad}}{\text{sec}} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} \times \frac{60 \text{ sec}}{1 \text{ min}}$$

$$\omega = \mathbf{6{,}105.54 \text{ rpm}}$$

Problem 140

At what angular velocity will the stress of the rotating steel ring equal 150 MPa if its mean radius is 220 mm? The density of steel 7.85 Mg/m^3.

Solution 140

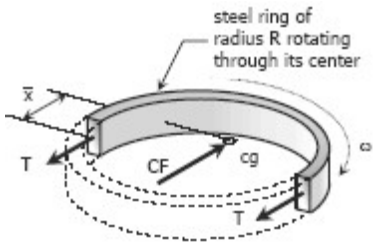
$CF = M\omega^2\bar{x}$

Where: $M = \rho V = \rho A \pi R$

$\bar{x} = 2R/\pi$

$CF = \rho A \pi R \omega^2 (2R/\pi)$

$CF = 2\rho A R^2 \omega^2$



FBD of Ring in Rotation

$2T = CF$

$2\sigma A = 2\rho A R^2 \omega^2$

$\sigma = \rho R^2 \omega^2$

From the given (Note: 1 N = 1 kg·m/sec^2):

σ = 150 MPa

= 150 000 000 kg·m/sec^2·m^2

= 150 000 000 kg/m·sec^2

ρ = 7.85 Mg/m^3 = 7850 kg/m^3

R = 220 mm = 0.22 m

$150\,000\,000 = 7850(0.22)^2\omega^2$

$\omega = 628.33$ rad/sec

Problem 141

The tank shown in Fig. P-141 is fabricated from 1/8-in steel plate. Calculate the maximum longitudinal and circumferential stress caused by an internal pressure of 125 psi.

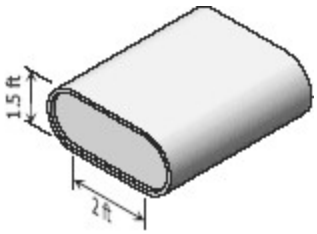
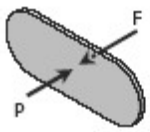


Figure P-141

Solution 141



See dimensions in Fig. P-141, thickness, t = 1/8 in.

Longitudinal Stress:

$F = pA = 125[1.5(2) + \frac{1}{4}\pi(1.5)^2](12)^2$

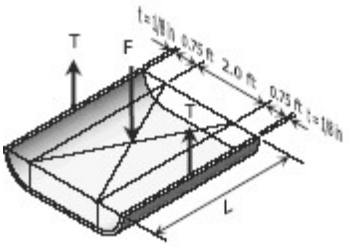
$F = 85\,808.62$ lbs

$P = F$

$\sigma_l [2(2 \times 12)(\frac{1}{8}) + \pi(1.5 \times 12)(\frac{1}{8})] = 85\,808.62$

$\sigma_l = 6\,566.02$ psi

$\sigma_l = 6.57$ ksi



Circumferential Stress:

$F = pA = 125[(2 \times 12)L + 2(0.75 \times 12)L]$

$F = 5250L$ lbs

$2T = F$

$2[\sigma_t (\frac{1}{8}) L] = 5250L$

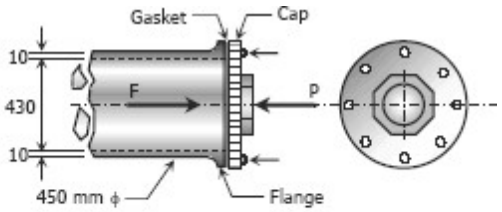
$\sigma_t = 21\,000$ psi

$\sigma_t = \mathbf{21\ ksi}$

Problem 142

A pipe carrying steam at 3.5 MPa has an outside diameter of 450 mm and a wall thickness of 10 mm. A gasket is inserted between the flange at one end of the pipe and a flat plate used to cap the end. How many 40-mm-diameter bolts must be used to hold the cap on if the allowable stress in the bolts is 80 MPa, of which 55 MPa is the initial stress? What circumferential stress is developed in the pipe? Why is it necessary to tighten the bolt initially, and what will happen if the steam pressure should cause the stress in the bolts to be twice the value of the initial stress?

Solution 142



$F = \sigma A$

$= 3.5[\frac{1}{4}\pi(430^2)]$

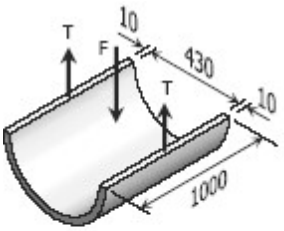
$= 508\,270.42$ N

$P = F$

$(\sigma_{bolt} A)n = 508\,270.42$ N

$(80 - 55)[\frac{1}{4}\pi(40^2)]n = 508\,270.42$

$n = 16.19$ say **17 bolts**



Circumferential stress (consider 1-m strip):

$F = pA = 3.5[430(1000)]$

$F = 1\,505\,000$ N

$2T = F$

$2[\sigma_t (1000)(10)] = 1\,505\,000$

$\sigma_t = 75.25$ MPa

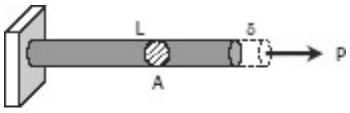
Discussion:

It is necessary to tighten the bolts initially to press the gasket to the flange, to avoid leakage of steam. If the pressure will cause 110 MPa of stress to each bolt causing it to fail, leakage will occur. If this is sudden, the cap may blow.

Strain

Simple Strain

Also known as unit deformation, strain is the ratio of the change in length caused by the applied force, to the original length.



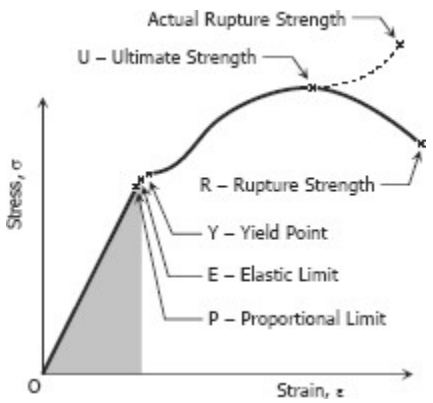
$$\varepsilon = \frac{\delta}{L}$$

where δ is the deformation and L is the original length, thus ε is dimensionless.

Stress-Strain Diagram

Suppose that a metal specimen be placed in tension-compression testing machine. As the axial load is gradually increased in increments, the total elongation over the gage length is measured at each increment of the load and this is continued until failure of the specimen takes place. Knowing the original cross-sectional area and length of the specimen, the normal stress σ and the strain ε can be obtained. The graph of these quantities with the stress σ along the y-axis and the strain ε along the x-axis is called the stress-strain diagram. The stress-strain diagram differs in form for various materials. The diagram shown below is that for a medium carbon structural steel.

Metallic engineering materials are classified as either ductile or brittle materials. A ductile material is one having relatively large tensile strains up to the point of rupture like structural steel and aluminum, whereas brittle materials has a relatively small strain up to the point of rupture like cast iron and concrete. An arbitrary strain of 0.05 mm/mm is frequently taken as the dividing line between these two classes.



PROPORTIONAL LIMIT (HOOKE'S LAW)

Robert Hooke

From the origin O to the point called proportional limit, the stress-strain curve is a straight line. This linear relation between elongation and the axial force causing was first noticed by Sir Robert Hooke in 1678 and is called Hooke's Law that within the proportional limit, the stress is directly proportional to strain or

$$\sigma \propto \varepsilon \text{ or } \sigma = k\,\varepsilon$$

The constant of proportionality k is called the Modulus of Elasticity E or Young's Modulus and is equal to the slope of the stress-strain diagram from O to P. Then

$$\sigma = E\,\varepsilon$$

ELASTIC LIMIT

The elastic limit is the limit beyond which the material will no longer go back to its original shape when the load is removed, or it is the maximum stress that may e developed such that there is no permanent or residual deformation when the load is entirely removed.

ELASTIC AND PLASTIC RANGES

The region in stress-strain diagram from O to P is called the elastic range. The region from P to R is called the plastic range.

YIELD POINT

Yield point is the point at which the material will have an appreciable elongation or yielding without any increase in load.

ULTIMATE STRENGTH

The maximum ordinate in the stress-strain diagram is the ultimate strength or tensile strength.

RAPTURE STRENGTH

Rapture strength is the strength of the material at rupture. This is also known as the breaking strength.

MODULUS OF RESILIENCE

Modulus of resilience is the work done on a unit volume of material as the force is gradually increased from O to P, in Nm/m^3. This may be calculated as the area under the stress-strain curve from the origin O to up to the elastic limit E (the shaded area in the figure). The resilience of the material is its ability to absorb energy without creating a permanent distortion.

MODULUS OF TOUGHNESS

Modulus of toughness is the work done on a unit volume of material as the force is gradually increased from O to R, in Nm/m^3. This may be calculated as the area under the entire stress-strain curve (from O to R). The toughness of a material is its ability to absorb energy without causing it to break.

WORKING STRESS, ALLOWABLE STRESS, AND FACTOR OF SAFETY

Working stress is defined as the actual stress of a material under a given loading. The maximum safe stress that a material can carry is termed as the allowable stress. The allowable stress should be limited to values not exceeding the proportional limit. However, since proportional limit is difficult to determine accurately, the allowable tress is taken as either the yield point or ultimate strength divided by a factor of safety. The ratio of this strength (ultimate or yield strength) to allowable strength is called the factor of safety.

AXIAL DEFORMATION

In the linear portion of the stress-strain diagram, the tress is proportional to strain and is given by

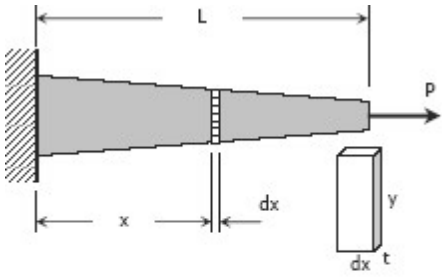
$$\sigma = E\varepsilon$$

since $\sigma = P / A$ and $\varepsilon e = \delta / L$, then $P / A = E \delta / L$. Solving for δ,

$$\delta = \frac{PL}{AE} = \frac{\sigma L}{E}$$

To use this formula, the load must be axial, the bar must have a uniform cross-sectional area, and the stress must not exceed the proportional limit. If however, the cross-sectional area is not uniform, the axial deformation can be determined by considering a differential length and applying integration.

If however, the cross-sectional area is not uniform, the axial deformation can be determined by considering a differential length and applying integration.



$$\delta = \frac{P}{E}\int_0^L \frac{dx}{A}$$

where A = ty and y and t, if variable, must be expressed in terms of x.

For a rod of unit mass ρ suspended vertically from one end, the total elongation due to its own weight is

$$\delta = \frac{\rho g L^2}{2E} = \frac{MgL}{2AE}$$

where ρ is in kg/m^3, L is the length of the rod in mm, M is the total mass of the rod in kg, A is the cross-sectional area of the rod in mm^2, and g = 9.81 m/s^2.

STIFFNESS, k

Stiffness is the ratio of the steady force acting on an elastic body to the resulting displacement. It has the unit of N/mm.

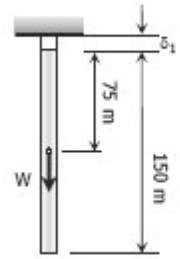
$$k = P / \delta$$

SOLVED PROBLEMS IN AXIAL DEFORMATION

Problem 206

A steel rod having a cross-sectional area of 300 mm^2 and a length of 150 m is suspended vertically from one end. It supports a tensile load of 20 kN at the lower end. If the unit mass of steel is 7850 kg/m^3 and E = 200 × 10^3 MN/m^2, find the total elongation of the rod.

Solution 206



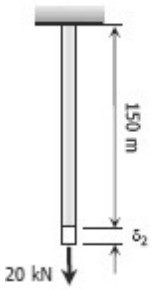
Let δ = total elongation
δ_1 = elongation due to its own weight
δ_2 = elongation due to applied load

$$\delta = \delta_1 + \delta_2$$

$$\delta_1 = \frac{PL}{AE}$$

Where: P = W = 7850(1/1000)3(9.81)[300(150)(1000)]
P = 3465.3825 N
L = 75(1000) = 75 000 mm
A = 300 mm^2
E = 200 000 MPa

$$\delta_1 = \frac{3465.3825\,(75000)}{300\,(200\,000)} = 4.33 \text{ mm}$$



$$\delta_2 = \frac{PL}{AE}$$

Where: P = 20 kN = 20 000 N
L = 150 m = 150 000 mm
A = 300 mm^2
E = 200 000 MPa

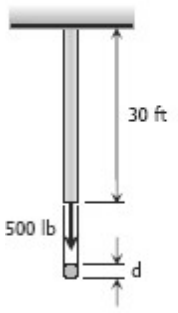
$$\delta_2 = \frac{20000(150000)}{300(200000)} = 50 \text{ mm}$$

Total elongation:
δ = 4.33 + 50 = **54.33 mm**

Problem 207

A steel wire 30 ft long, hanging vertically, supports a load of 500 lb. Neglecting the weight of the wire, determine the required diameter if the stress is not to exceed 20 ksi and the total elongation is not to exceed 0.20 in. Assume E = 29 × 10^6 psi.

Solution 207



Based on maximum allowable stress:

$$\sigma = \frac{P}{A}$$

$$20\,000 = \frac{500}{\frac{1}{4}\pi d^2}$$

$$d = 0.0318 \text{ in}$$

Based on maximum allowable deformation:

$$\delta = \frac{PL}{AE}$$

$$0.20 = \frac{500(30\times12)}{\frac{1}{4}\pi d^2(29\times10^6)}$$

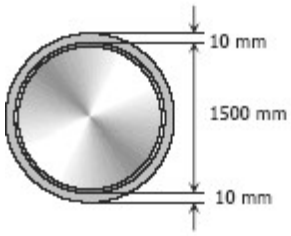
$$d = 0.0395 \text{ in}$$

Use the bigger diameter, $d = \mathbf{0.0395\ in}$

Problem 208

A steel tire, 10 mm thick, 80 mm wide, and 1500.0 mm inside diameter, is heated and shrunk onto a steel wheel 1500.5 mm in diameter. If the coefficient of static friction is 0.30, what torque is required to twist the tire relative to the wheel? Neglect the deformation of the wheel. Use E = 200 GPa.

Solution 208



$$\delta = \frac{PL}{AE}$$

Where: $\delta = \pi(1500.5 - 1500) = 0.5\pi$ mm
$P = T$
$L = 1500\pi$ mm
$A = 10(80) = 800$ mm^2
$E = 200\,000$ MPa

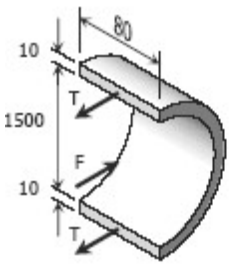
$$0.5\pi = \frac{T(1500\pi)}{800(200\,000)}$$

$$T = 53\,333.33 \text{ N}$$

$$F = 2T$$

$$p\,(1500)(80) = 2(53\,333.33)$$

$$p = 0.8889 \text{ MPa} \quad \rightarrow \text{internal pressure}$$



Total normal force, N:

$N = p \times$ contact area between tire and wheel

$$N = 0.8889 \times \pi(1500.5)(80)$$

$$N = 335\,214.92 \text{ N}$$

Friction resistance, f:

$$f = \mu N = 0.30(335\,214.92)$$

$$f = 100\,564.48 \text{ N} = 100.56 \text{ kN}$$

Torque = $f \times \frac{1}{2}$ (diameter of wheel)

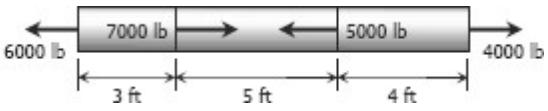
Torque = 100.56×0.75025

Torque = **75.44 kN·m**

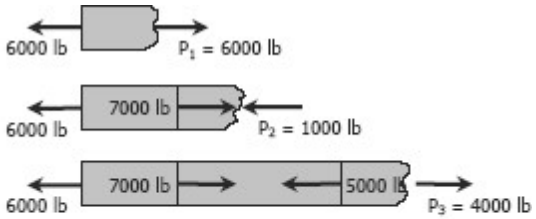
Problem 209

An aluminum bar having a cross-sectional area of 0.5 in^2 carries the axial loads applied at the positions shown in Fig. P-209. Compute the total change in length of the bar if E = 10 × 10^6 psi. Assume the bar is suitably braced to prevent lateral buckling.

Figure P-209 and P-210



Solution 209



$P_1 = 6000$ lb tension
$P_2 = 1000$ lb compression
$P_3 = 4000$ lb tension

$$\delta = \frac{PL}{AE}$$

$$\delta = \delta_1 - \delta_2 + \delta_3$$

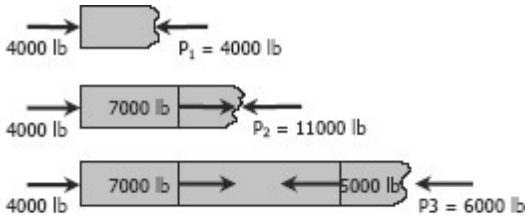
$$\delta = \frac{6000(3\times12)}{0.5(10\times10^6)} - \frac{1000(5\times12)}{0.5(10\times10^6)} + \frac{4000(4\times12)}{0.5(10\times10^6)}$$

$\delta = 0.0696$ in (lengthening)

Problem 210

Solve Prob. 209 if the points of application of the 6000-lb and the 4000-lb forces are interchanged.

Solution 210



$P_1 = 4000$ lb compression
$P_2 = 11000$ lb compression
$P_3 = 6000$ lb compression

$$\delta = \frac{PL}{AE}$$

$$\delta = -\delta_1 - \delta_2 - \delta_3$$

$$\delta = -\frac{4000(3\times12)}{0.5(10\times10^6)} - \frac{11000(5\times12)}{0.5(10\times10^6)} - \frac{6000(4\times12)}{0.5(10\times10^6)}$$

$\delta = -0.19248$ in = **0.19248 in (shortening)**

Problem 211

A bronze bar is fastened between a steel bar and an aluminum bar as shown in Fig. P-211. Axial loads are applied at the positions indicated. Find the largest value of P that will not exceed an overall deformation of 3.0 mm, or the following stresses: 140 MPa in the steel, 120 MPa in the bronze, and 80 MPa in the aluminum. Assume that the assembly is suitably braced to prevent buckling. Use E_{st} = 200 GPa, E_{al} = 70 GPa, and E_{br} = 83 GPa.

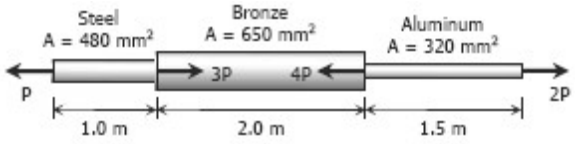


Figure P -211

Solution 211

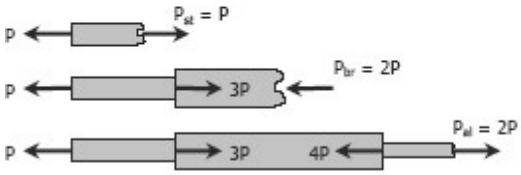
Based on allowable stresses:

Steel:

$P_{st} = \sigma_{st} A_{st}$

$P = 140(480) = 67\,200$ N

$P = 67.2$ kN



Bronze:

$P_{br} = \sigma_{br} A_{br}$

$2P = 120(650) = 78\,000$

$P = 39\,000$ N $= 39$ kN

Aluminum:

$P_{al} = \sigma_{al} A_{al}$

$2P = 80(320) = 25\,600$ N

$P = 12\,800$ N $= 12.8$ kN

Based on allowable deformation:
(steel and aluminum lengthens, bronze shortens)

$\delta = \delta_{st} - \delta_{br} + \delta_{al}$

$$3 = \frac{P(1000)}{480(200\,000)} - \frac{2P(2000)}{650(70\,000)} + \frac{2P(1500)}{320(83\,000)}$$

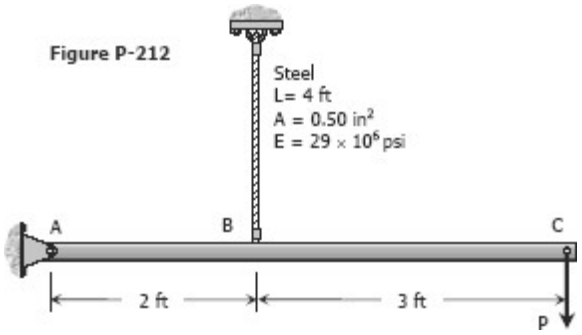
$$3 = \left(\frac{1}{96000} - \frac{1}{11375} + \frac{3}{26560}\right)P$$

$P = 84\,610.99$ N $= 84.61$ kN

Use the smallest value of P, $P = 12.8$ kN

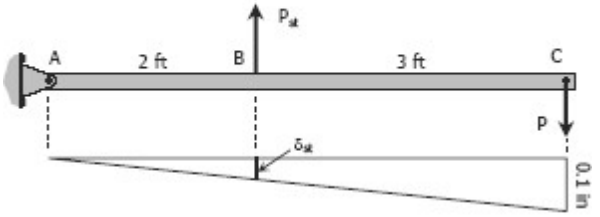
Problem 212

The rigid bar ABC shown in Fig. P-212 is hinged at A and supported by a steel rod at B. Determine the largest load P that can be applied at C if the stress in the steel rod is limited to 30 ksi and the vertical movement of end C must not exceed 0.10 in.



Solution 212

Free body and deformation diagrams:



Based on maximum stress of steel rod:

$\Sigma M_A = 0$

$5P = 2P_{st}$

$P = 0.4P_{st}$

$P = 0.4\sigma_{st}A_{st}$

$P = 0.4[30(0.50)]$

$P = 6$ kips

Based on movement at C:

$$\frac{\delta_{st}}{2} = \frac{0.1}{5}$$

$\delta_{st} = 0.04$ in

$$\frac{P_{st}L}{AE} = 0.04$$

$$\frac{P_{st}(4\times12)}{0.50(29\times10^6)} = 0.04$$

$P_{st} = 12\,083.33$ lb

$\Sigma M_A = 0$

$5P = 2P_{st}$

$P = 0.4P_{st}$

$P = 0.4(12\,083.33)$

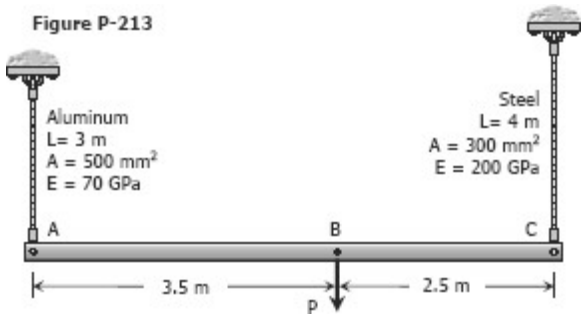
$P = 4833.33$ lb $= 4.83$ kips

Use the smaller value, $P = 4.83$ kips

Problem 213

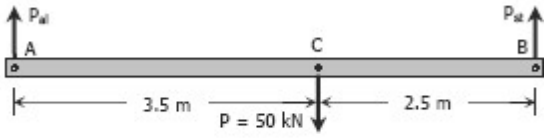
The rigid bar AB, attached to two vertical rods as shown in Fig. P-213, is horizontal before the load P is applied. Determine the vertical movement of P if its magnitude is 50 kN.

Figure P-213



Solution 213

Free body diagram:



For aluminum:

$[\Sigma M_B = 0]$ $\quad 6P_{al} = 2.5(50)$

$P_{al} = 20.83$ kN

$\left[\delta = \frac{PL}{AE}\right]_{al}$ $\quad \delta_{al} = \frac{20.83(3)1000^2}{500(70000)}$

$\delta_{al} = 1.78$ mm

For steel:

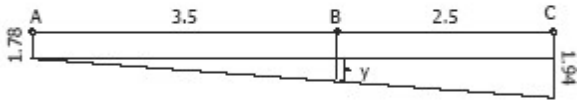
$[\Sigma M_A = 0]$ $\quad 6P_{st} = 3.5(50)$

$P_{st} = 29.17$ kN

$\left[\delta = \frac{PL}{AE}\right]_{st}$ $\quad \delta_{st} = \frac{29.17(4)1000^2}{300(200000)}$

$\delta_{st} = 1.94$ mm

Movement diagram:



$\frac{y}{3.5} = \frac{1.94 - 1.78}{6}$

$y = 0.09$ mm

δ_B = vertical movement of P

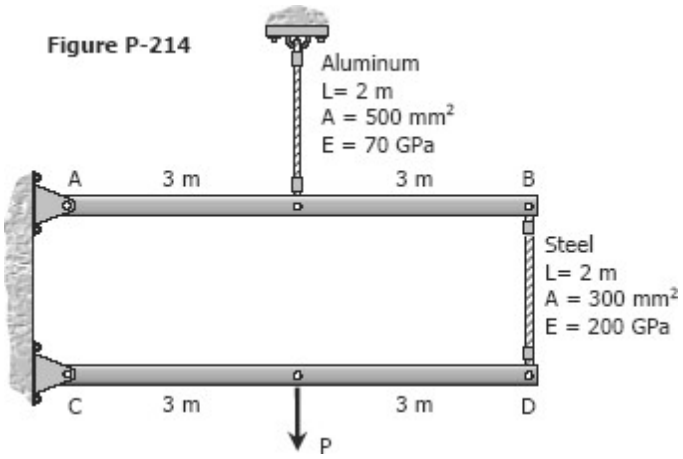
$\delta_B = 1.78 + y = 1.78 + 0.09$

$\delta_B = 1.87$ mm

Problem 214

The rigid bars AB and CD shown in Fig. P-214 are supported by pins at A and C and the two rods. Determine the maximum force P that can be applied as shown if its vertical movement is limited to 5 mm. Neglect the weights of all members.

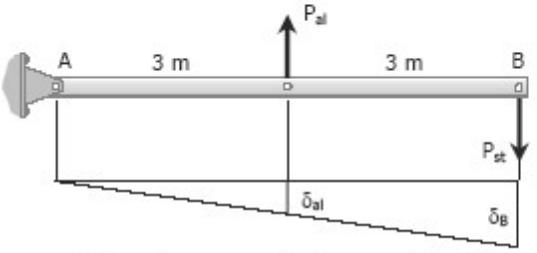
Figure P-214



Solution 214

$[\sum M_A = 0]$ $\quad 3P_{al} = 6P_{st}$

$P_{al} = 2P_{st}$



FBD and movement diagram of bar AB

By ratio and proportion:

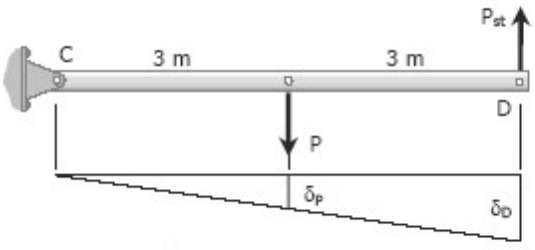
$$\frac{\delta_B}{6} = \frac{\delta_{al}}{3}$$

$$\delta_B = 2\delta_{al} = 2\left[\frac{PL}{AE}\right]_{al}$$

$$\delta_B = 2\left[\frac{P_{al}(2000)}{500(70\,000)}\right]$$

$$\delta_B = \tfrac{1}{8750} P_{al} = \tfrac{1}{8750} (2P_{st})$$

$$\delta_B = \tfrac{1}{4375} P_{st} \rightarrow \text{movement of B}$$



FBD and movement diagram of bar CD

Movement of D:

$$\delta_D = \delta_{st} + \delta_B = \left[\frac{PL}{AE}\right]_{st} + \tfrac{1}{4375} P_{st}$$

$$\delta_D = \frac{P_{st}(2000)}{300(200\,000)} + \tfrac{1}{4375} P_{st}$$

$$\delta_D = \tfrac{11}{42000} P_{st}$$

$[\sum M_C = 0]$ $\quad 6P_{st} = 3P$

$P_{st} = \tfrac{1}{2} P$

By ratio and proportion:

$$\frac{\delta_P}{3} = \frac{\delta_D}{6}$$

$$\delta_P = \tfrac{1}{2} \delta_D = \tfrac{1}{2} \left(\tfrac{11}{42000} P_{st}\right)$$

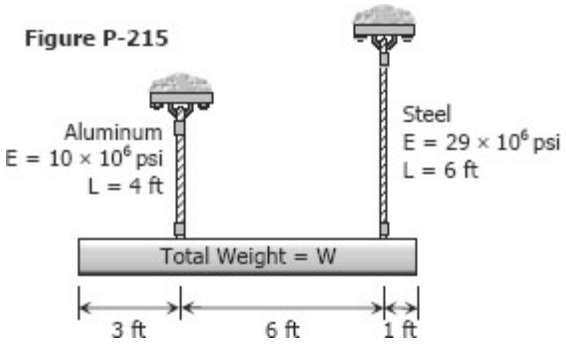
$$\delta_P = \tfrac{11}{84000} P_{st}$$

$$5 = \tfrac{11}{84000} \left(\tfrac{1}{2} P\right)$$

$P = 76\,363.64$ N = **76.4 kN**

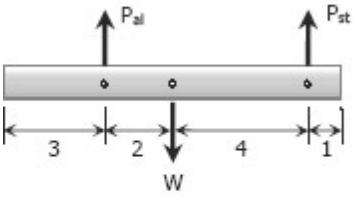
Problem 215

A uniform concrete slab of total weight W is to be attached, as shown in Fig. P-215, to two rods whose lower ends are on the same level. Determine the ratio of the areas of the rods so that the slab will remain level.



Solution 215

$$[\sum M_{al} = 0] \qquad 6P_{st} = 2W$$
$$P_{st} = \tfrac{1}{3}W$$



$$[\sum M_{st} = 0] \qquad 6P_{al} = 4W$$
$$P_{al} = \tfrac{2}{3}W$$

$$\delta_{st} = \delta_{al}$$

$$\left[\frac{PL}{AE}\right]_{st} = \left[\frac{PL}{AE}\right]_{al}$$

$$\frac{\frac{1}{3}W(6\times12)}{A_{st}(29\times10^6)} = \frac{\frac{2}{3}W(4\times12)}{A_{al}(10\times10^6)}$$

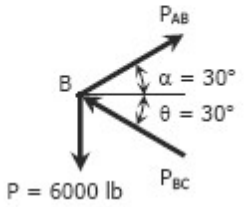
$$\frac{A_{al}}{A_{st}} = \frac{\frac{2}{3}W(4\times12)(29\times10^6)}{\frac{1}{3}W(6\times12)(10\times10^6)}$$

$$A_{al}/A_{st} = 3.867$$

Problem 216

As shown in Fig. P-216, two aluminum rods AB and BC, hinged to rigid supports, are pinned together at B to carry a vertical load P = 6000 lb. If each rod has a crosssectional area of 0.60 in^2 and E = 10 × 10^6 psi, compute the elongation of each rod and the horizontal and vertical displacements of point B. Assume α = 30° and θ = 30°.

Solution 216



$[\sum F_H = 0]$ $P_{AB} \cos 30° = P_{BC} \cos 30°$

$P_{AB} = P_{BC}$

$[\sum F_V = 0]$ $P_{AB} \sin 30° + P_{BC} \sin 30° = 6000$

$P_{AB}\,(0.5) + P_{AB}\,(0.5) = 6000$

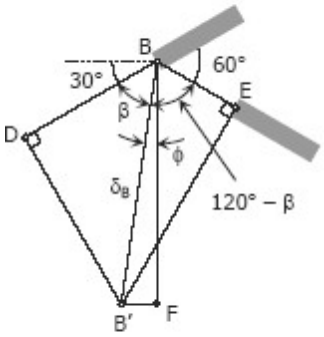
$P_{AB} = 6000$ lb tension

$P_{BC} = 6000$ lb compression

$$\delta = \frac{PL}{AE}$$

$$\delta_{AB} = \frac{6000(10\times 12)}{0.6(10\times 10^6)} = \textbf{0.12 in.}\ \text{lengthening}$$

$$\delta_{BC} = \frac{6000(6\times 12)}{0.6(10\times 10^6)} = \textbf{0.072 in.}\ \text{shortening}$$



Movement of B

$DB = \delta_{AB} = 0.12$ in

$BE = \delta_{BE} = 0.072$ in

$\delta_B = BB'$ = displacement of B

B' = final position of B after elongation

Triangle BDB':

$$\cos\beta = \frac{0.12}{\delta_B}$$

$$\delta_B = \frac{0.12}{\cos\beta}$$

Triangle BEB':

$$\cos(120° - \beta) = \frac{0.072}{\delta_B}$$

$$\delta_B = \frac{0.072}{\cos(120° - \beta)}$$

$\delta_B = \delta_B$

$$\frac{0.12}{\cos\beta} = \frac{0.072}{\cos(120° - \beta)}$$

$$\frac{\cos 120° \cos \beta + \sin 120° \sin \beta}{\cos \beta} = 0.6$$

$-0.5 + \sin 120° \tan \beta = 0.6$

$\tan \beta = 1.1/\sin 120°;\ \beta = 51.79°$

$\phi = 90 - (30° + \beta) = 90° - (30° + 51.79°)$

$\phi = 8.21°$

$$\delta_B = \frac{0.12}{\cos 51.79°}$$

$\delta_B = 0.194$ in

Triangle BFB':

$\delta_h = B'F = \delta_B \sin \phi = 0.194 \sin 8.21°$

$\delta_h = 0.0277$ in

$\delta_h = \mathbf{0.0023\ ft}$ → horizontal displacement of B

$\delta_v = BF = \delta_B \cos \phi = 0.194 \cos 8.21°$

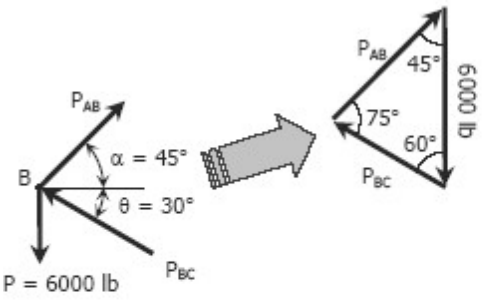
$\delta_v = 0.192$ in

$\delta_v = \mathbf{0.016\ ft}$ → vertical displacement of B

Problem 217

Solve Prob. 216 if rod AB is of steel, with E = 29 × 10^6 psi. Assume $\alpha = 45°$ and $\theta = 30°$; all other data remain unchanged.

Solution 217



By Sine Law

$$\frac{P_{AB}}{\sin 60°} = \frac{6000}{\sin 75°}$$

$P_{AB} = 5379.45$ lb (Tension)

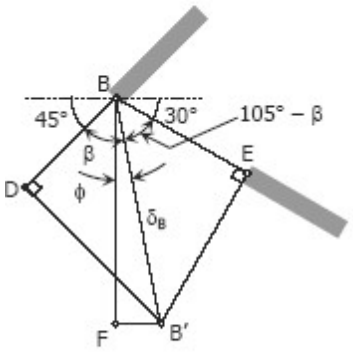
$$\frac{P_{BC}}{\sin 45°} = \frac{6000}{\sin 75°}$$

$P_{BC} = 4392.30$ lb (Compression)

$$\delta = \frac{PL}{AE}$$

$$\delta_{AB} = \frac{5379.45(10 \times 12)}{0.6(29 \times 10^6)} = \mathbf{0.0371\ in.}\ \text{(lengthening)}$$

$$\delta_{BC} = \frac{4392.30(6\times12)}{0.6(10\times10^6)} = \mathbf{0.0527\ in.}\ \text{(shortening)}$$



Movement of B

$DB = \delta_{AB} = 0.0371$ in
$BE = \delta_{BE} = 0.0527$ in
$\delta_B = BB'$ = displacement of B
B' = final position of B after deformation

Triangle BDB':

$$\cos\beta = \frac{0.0371}{\delta_B}$$

$$\delta_B = \frac{0.0371}{\cos\beta}$$

Triangle BEB':

$$\cos(105° - \beta) = \frac{0.0527}{\delta_B}$$

$$\delta_B = \frac{0.0527}{\cos(105° - \beta)}$$

$$\delta_B = \delta_B$$

$$\frac{0.0371}{\cos\beta} = \frac{0.0527}{\cos(105° - \beta)}$$

$$\frac{\cos 105°\cos\beta + \sin 105°\sin\beta}{\cos\beta} = 1.4205$$

$$-0.2588 + 0.9659\tan\beta = 1.4205$$

$$\tan\beta = \frac{1.4205 + 0.2588}{0.9659}$$

$$\tan\beta = 1.7386$$

$$\beta = 60.1°$$

$$\delta_B = \frac{0.0371}{\cos 60.1°}$$

$$\delta_B = 0.0744\ \text{in}$$

$$\phi = (45° + \beta) - 90°$$
$$= (45° + 60.1°) - 90°$$
$$= 15.1°$$

Triangle BFB':

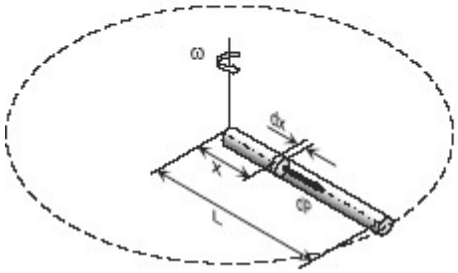
$\delta_h = FB' = \delta_B \sin\phi = 0.0744 \sin 15.1°$
$\delta_h = 0.0194$ in
$\delta_h = \mathbf{0.00162\ ft}$ → horizontal displacement of B

$\delta_v = BF = \delta_B \cos\phi = 0.0744 \cos 15.1°$
$\delta_v = 0.07183$ in
$\delta_v = \mathbf{0.00598\ ft}$ → vertical displacement of B

Problem 218

A uniform slender rod of length L and cross sectional area A is rotating in a horizontal plane about a vertical axis through one end. If the unit mass of the rod is ρ, and it is rotating at a constant angular velocity of ω rad/sec, show that the total elongation of the rod is $\rho\omega^2L^3/3E$.

Solution 218



$$\delta = \frac{PL}{AE}$$

from the frigure:

$$d\delta = \frac{dP\ x}{AE}$$

Where:
dP = centrifugal force of differential mass
$dP = dM\ \omega^2 x = (\rho A\ dx)\omega^2\ x$
$dP = \rho A\omega^2\ x\ dx$

$$d\delta = \frac{(\rho A\omega^2 x\ dx)\ x}{AE}$$

$$\delta = \frac{\rho\omega^2}{E}\int_0^L x^2\ dx\ = \frac{\rho\omega^2}{E}\left[\frac{x^3}{3}\right]_0^L$$

$$\delta = \frac{\rho\omega^2}{3E}\left[L^3 - 0^3\right]$$

$\delta = \rho\omega^2L^3/3E$ *ok!*

Problem 219

A round bar of length L, which tapers uniformly from a diameter D at one end to a smaller diameter d at the other, is suspended vertically from the large end. If w is the weight per unit volume, find the elongation of the rod caused by its own weight. Use this result to determine the elongation of a cone suspended from its base.

Solution 219

$$\delta = \frac{PL}{AE}$$

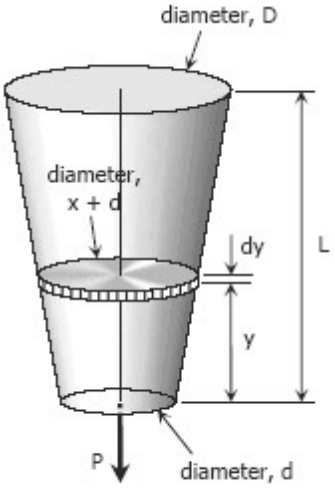
For the differential strip shown:

$\delta = d\delta$

P = weight carried by the strip
= weight of segment y

L = dy

A = area of the strip



For weight of segment y (Frustum of a cone):

$$P = wV_y$$

From section along the axis

$$\frac{x}{y} = \frac{D-d}{L}$$

$$x = \frac{D-d}{L}y$$

Volume for frustum of cone

$$V = \tfrac{1}{3}\pi h\,(R^2 + r^2 + Rr)$$

$$V_y = \tfrac{1}{3}\pi h\,[\tfrac{1}{4}(x+d)^2 + \tfrac{1}{4}d^2 + \tfrac{1}{2}(x+d)(\tfrac{1}{2}d)]$$

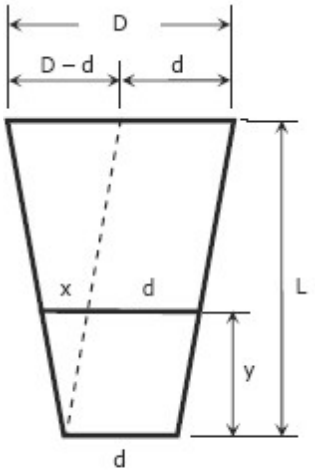
$$V_y = \tfrac{1}{12}\pi y\,[(x+d)^2 + d^2 + (x+d)d]$$

$$P = \tfrac{1}{12}\pi w\,[(x+d)^2 + d^2 + (x+d)d]\,y$$

$$P = \tfrac{1}{12}\pi w\,[x^2 + 2xd + d^2 + d^2 + xd + d^2]\,y$$

$$P = \tfrac{1}{12}\pi w\,[x^2 + 3xd + 3d^2]\,y$$

$$P = \frac{\pi w}{12}\left[\frac{(D-d)^2}{L^2}y^2 + \frac{3d(D-d)}{L}y + 3d^2\right]y$$



Section along the axis of the bar

Area of the strip:

$$A = \tfrac{1}{4}\pi(x+d)^2 = \frac{\pi}{4}\left(\frac{D-d}{L}y + d\right)^2$$

Thus,

$$\delta = \frac{PL}{AE}$$

$$d\delta = \frac{\frac{\pi w}{12}\left[\frac{(D-d)^2}{L^2}y^2 + \frac{3d(D-d)}{L}y + 3d^2\right]y\,dy}{\frac{\pi}{4}\left(\frac{D-d}{L}y + d\right)^2 E}$$

$$d\delta = \frac{4w}{12E}\left[\frac{\frac{(D-d)^2}{L^2}y^2 + \frac{3d(D-d)}{L}y + 3d^2}{\frac{(D-d)^2}{L^2}y^2 + \frac{2d(D-d)}{L}y + d^2}\right]y\,dy$$

$$d\delta = \frac{w}{3E}\left[\frac{\frac{(D-d)^2y^2 + 3Ld(D-d)y + 3L^2d^2}{L^2}}{\frac{(D-d)^2y^2 + 2Ld(D-d)y + L^2d^2}{L^2}}\right]y\,dy$$

$$d\delta = \frac{w}{3E}\left[\frac{(D-d)^2y^2 + 3Ld(D-d)y + 3L^2d^2}{(D-d)^2y^2 + 2Ld(D-d)y + L^2d^2}\right]y\,dy$$

Let: a = D – d; b = Ld

$$d\delta = \frac{w}{3E}\left[\frac{a^2y^2 + 3aby + 3b^2}{a^2y^2 + 2aby + b^2}\right]y\,dy$$

$$d\delta = \frac{w}{3E}\left[\frac{a^2y^2 + 3aby + 3b^2}{(ay)^2 + 2(ay)b + b^2} \times \frac{a}{a}\right]y\,dy$$

$$d\delta = \frac{w}{3aE}\left[\frac{a^3y^3 + 3(a^2y^2)b + 3(ay)b^2}{(ay+b)^2}\right]dy$$

$$d\delta = \frac{w}{3aE}\left\{\frac{[(ay)^3 + 3(ay)^2b + 3(ay)b^2 + b^3] - b^3}{(ay+b)^2}\right\}dy$$

The quantity $(ay)^3 + 3(ay)^2b + 3(ay)b^2 + b^3$ is the expansion of $(ay + b)^3$

$$d\delta = \frac{w}{3aE}\left[\frac{(ay+b)^3 - b^3}{(ay+b)^2}\right]dy$$

$$d\delta = \frac{w}{3aE}\left[\frac{(ay+b)^3}{(ay+b)^2} - \frac{b^3}{(ay+b)^2}\right]dy$$

$$d\delta = \frac{w}{3aE}[(ay+b)-b^3(ay+b)^{-2}]\,dy$$

$$\delta = \frac{w}{3aE}\int_0^L [(ay+b)-b^3(ay+b)^{-2}]\,dy$$

$$\delta = \frac{w}{3aE}\left[\frac{(ay+b)^2}{2a}-\frac{b^3(ay+b)^{-1}}{-a}\right]_0^L$$

$$\delta = \frac{w}{3a^2E}\left[\frac{(ay+b)^2}{2}+\frac{b^3}{ay+b}\right]_0^L$$

$$\delta = \frac{w}{3a^2E}\left\{\left[\tfrac{1}{2}(aL+b)^2+\frac{b^3}{aL+b}\right]-\left[\tfrac{1}{2}b^2+\frac{b^3}{b}\right]\right\}$$

$$\delta = \frac{w}{3a^2E}\left\{\tfrac{1}{2}(aL+b)^2+\frac{b^3}{aL+b}-\tfrac{3}{2}b^2\right\}$$

$$\delta = \frac{w}{3a^2E}\left[\frac{(aL+b)^3+2b^3-3b^2(aL+b)}{2(aL+b)}\right]$$

$$\delta = \frac{w}{6a^2E}\left[\frac{(aL)^3+3(aL)^2b+3(aL)b^2+b^3+2b^3-3ab^2L-3b^3}{aL+b}\right]$$

$$\delta = \frac{w}{6a^2E}\left[\frac{a^3L^3+3a^2bL^2}{aL+b}\right]$$;Note: a = D – d & b = Ld

$$\delta = \frac{w}{6(D-d)^2E}\left[\frac{(D-d)^3L^3+3(D-d)^2(Ld)L^2}{(D-d)L+(Ld)}\right]$$

$$\delta = \frac{w}{6(D-d)^2E}\left\{\frac{(D-d)L^3[(D-d)^2+3d(D-d)]}{LD-Ld+Ld}\right\}$$

$$\delta = \frac{wL^3}{6(D-d)E}\left[\frac{(D-d)^2+3d(D-d)}{LD}\right]$$

$$\delta = \frac{wL^3}{6(D-d)E}\left[\frac{D^2-2Dd+d^2+3Dd-3d^2}{LD}\right]$$

$$\delta = \frac{wL^3}{6(D-d)E}\left[\frac{D^2+Dd-2d^2}{LD}\right]$$

$$\delta = \frac{wL^3}{6(D-d)E}\left[\frac{D(D+d)-2d^2}{LD}\right]$$

$$\delta = \frac{wL^3}{6(D-d)E}\left[\frac{D(D+d)}{LD}\right]-\frac{wL^3}{6(D-d)E}\left[\frac{2d^2}{LD}\right]$$

$$\delta = \frac{wL^2(D+d)}{6E(D-d)}-\frac{wL^2d^2}{3ED(D-d)}$$

For a cone:

$D = D$ and $d = 0$

$$\delta = \frac{wL^2\cancel{(D+0)}}{6E\cancel{(D-0)}}-\frac{\cancel{wL^2(0)^2}}{3ED(D-0)}$$

$$\delta = \frac{wL^2}{6E}$$

SOLVED PROBLEMS IN STRAIN AND AXIAL DEFORMATION

Problem 203

The following data were recorded during the tensile test of a 14-mm-diameter mild steel rod. The gage length was 50 mm.

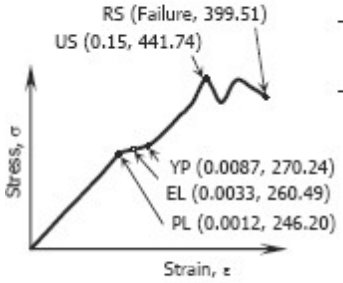
Load (N)	Elongation (mm)	Load (N)	Elongation (mm)
0	0	46200	1.25
6310	0.010	52400	2.50
12600	0.020	58500	4.50
18800	0.030	68000	7.50
25100	0.040	59000	12.50
31300	0.050	67800	15.50
37900	0.060	65000	20.00
40100	0.163	61500	Fracture
41600	0.433		

Plot the stress-strain diagram and determine the following mechanical properties: (a) proportional limits; (b) modulus of elasticity; (c) yield point; (d) ultimate strength; and (e) rupture strength.

Solution 203

Area, $A = \frac{1}{4}\pi(14)^2 = 49\pi$ mm^2; Length, $L = 50$ mm

Strain = Elongation/Length; Stress = Load/Area



Stress-Strain Diagram
(not drawn to scale)

PL = Proportional Limit
EL = Elastic Limit
YP = Yield Point
US = Ultimate Strength
RS = Rupture Strength

Load (N)	Elongation (mm)	Strain (mm/mm)	Stress (MPa)
0	0	0	0
6310	0.010	0.0002	40.99
12600	0.020	0.0004	81.85
18800	0.030	0.0006	122.13
25100	0.040	0.0008	163.05
31300	0.050	0.001	203.33
37900	0.060	0.0012	246.20
40100	0.163	0.0033	260.49
41600	0.433	0.0087	270.24
46200	1.250	0.025	300.12
52400	2.500	0.05	340.40
58500	4.500	0.09	380.02
68000	7.500	0.15	441.74
59000	12.500	0.25	383.27
67800	15.500	0.31	440.44
65000	20.000	0.4	422.25
61500	Failure		399.51

From stress-strain diagram:

(a) Proportional Limit = **246.20 MPa**

(b) Modulus of Elasticity

E = slope of stress-strain diagram within proportional limit

$$E = \frac{246.20}{0.0012} = 205\,166.67 \text{ MPa}$$

$E = 205.2$ GPa

(c) Yield Point = **270.24 MPa**

(d) Ultimate Strength = **441.74 MPa**

(e) Rupture Strength = **399.51 MPa**

Problem 204

The following data were obtained during a tension test of an aluminum alloy. The initial diameter of the test specimen was 0.505 in. and the gage length was 2.0 in.

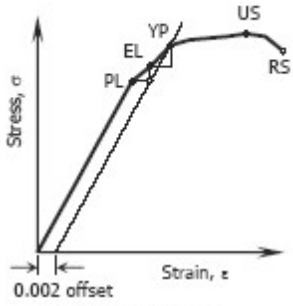
Load (lb)	Elongation (in.)	Load (lb)	Elongation (in.)
0	0	14 000	0.020
2 310	0.00220	14 400	0.025
4 640	0.00440	14 500	0.060
6 950	0.00660	14 600	0.080
9 290	0.00880	14 800	0.100
11 600	0.0110	14 600	0.120
12 600	0.0150	13 600	Fracture

Plot the stress-strain diagram and determine the following mechanical properties: (a) proportional limit; (b) modulus of elasticity; (c) yield point; (d) yield strength at 0.2% offset; (e) ultimate strength; and (f) rupture strength.

Solution 204

Area = $\frac{1}{4}\pi(0.505)^2 = 0.0638\pi$ in^2; Length, L = 2.0 in.
Strain = Elongation/Length; Stress = Load/Area

Load (lb)	Elongation (in)	Strain (in/in)	Stress (psi)
0	0	0	0
2310	0.0022	0.0011	11532.92
4640	0.0044	0.0022	23165.70
6950	0.0066	0.0033	34698.62
9290	0.0088	0.0044	46381.32
11600	0.011	0.0055	57914.24
12600	0.015	0.0075	62906.85
14000	0.02	0.01	69896.49
14400	0.025	0.0125	71893.54
14500	0.06	0.03	72392.80
14600	0.08	0.04	72892.06
14800	0.1	0.05	73890.58
14600	0.12	0.06	72892.06
13600	Fracture		67899.45



PL (0.0055, 57914.24)
EL (0.0075, 62906.85)
YP (0.01, 69896.49)
US (0.05, 73890.58)
RS (Failure, 67899.45)

From stress-strain diagram:

(a) Proportional Limit = **57,914.24 psi**

(b) Modulus of Elasticity:

$$E = \frac{57914.24}{0.0055} = 10{,}529{,}861.82 \text{ psi}$$

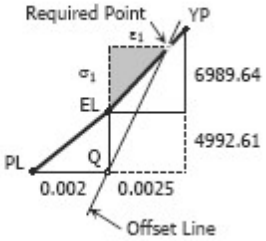
E = 10,529.86 ksi

(c) Yield Point = **69,896.49 psi**

(d) Yield Strength at 0.2% Offset:

Strain of Elastic Limit
= ε at PL + 0.002
= 0.0055 + 0.002
= 0.0075 in/in

The offset line will pass through Q(See figure):



Slope of 0.2% offset
= E = 10,529,861.82 psi

Test for location

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$10{,}529{,}861.82 = \frac{6989.64 + 4992.61}{\text{run}}$$

run = 0.00113793 < 0.0025, therefore, the required point is just before *YP*.

Slope of *EL* to *YP*

$$\frac{\sigma_1}{\varepsilon_1} = \frac{6989.64}{0.0025}$$

$$\frac{\sigma_1}{\varepsilon_1} = 2\,795\,856$$

$$\varepsilon_1 = \frac{\sigma_1}{2\,795\,856}$$

For required point

$$E = \frac{4992.61 + \sigma_1}{\varepsilon_1}$$

$$10\,529\,861.82 = \frac{4992.61 + \sigma_1}{\frac{\sigma_1}{2\,795\,856}}$$

$$3.7662\sigma_1 = 4992.61 + \sigma_1$$

$$\sigma_1 = 1804.84 \text{ psi}$$

Yield Strength at 0.2% Offset

$= EL + \sigma_1$

$= 62906.85 + 1804.84$

$= \mathbf{64{,}711.69 \text{ psi}}$

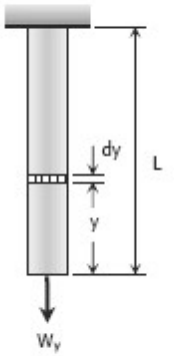
(e) Ultimate Strength = **73,890.58 psi**

(f) Rupture Strength = **67,899.45 psi**

Problem 205

A uniform bar of length L, cross-sectional area A, and unit mass ρ is suspended vertically from one end. Show that its total elongation is $\delta = \rho g L^2 / 2E$. If the total mass of the bar is M, show also that $\delta = MgL/2AE$.

Solution 205



$$\delta = \frac{PL}{AE}$$

From the figure:
$\delta = d\delta$
$P = Wy = (\rho Ay)g$
$L = dy$

$$d\delta = \frac{(\rho Ay)g\,dy}{AE}$$

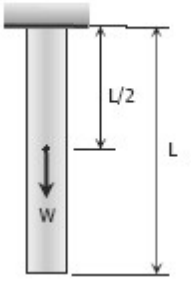
$$\delta = \frac{\rho g}{E}\int_0^L y\,dy = \frac{\rho g}{E}\left[\frac{y^2}{2}\right]_0^L$$

$$\delta = \frac{\rho g}{2E}\left[L^2 - 0^2\right] = \rho g L^2/2E \qquad ok!$$

Given the total mass M:
$\rho = M/V = M/AL$

$\delta = \rho g L^2/2E = (M/AL)(gL^2/2E)$
$\delta = MgL/2AE \qquad ok!$

Another Solution:



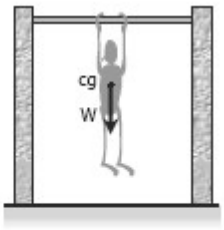
The weight will act at the center of gravity of the bar:

$$\delta = \frac{PL}{AE}$$

Where: $P = W = (\rho AL)g$
$L = L/2$

$$\delta = \frac{[(\rho AL)g](L/2)}{AE}$$

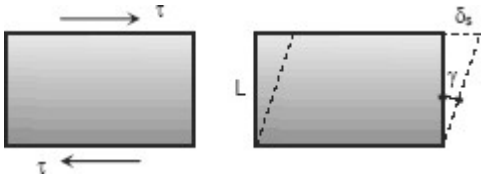
$$\delta = \frac{\rho g L^2}{2E} \qquad ok!$$



For you to feel the situation, position yourself in pull-up exercise with your hands on the bar and your body hang freely above the ground. Notice that your arms suffer all your weight and your lower body fells no stress (center of weight is approximately just below the chest). If your body is the bar, the elongation will occur at the upper half of it.

Shearing Deformation

Shearing forces cause shearing deformation. An element subject to shear does not change in length but undergoes a change in shape.



The change in angle at the corner of an original rectangular element is called the **shear strain** and is expressed as

$$\gamma = \frac{\delta_s}{L}$$

The ratio of the shear stress τ and the shear strain γ is called the **modulus of elasticity** in shear or modulus of rigidity and is denoted as G, in MPa.

$$G = \frac{\tau}{\gamma}$$

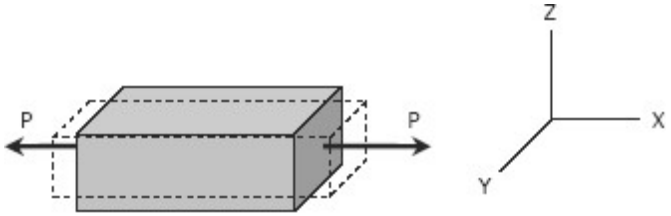
The relationship between the shearing deformation and the applied shearing force is

$$\delta_s = \frac{VL}{A_s G} = \frac{\tau L}{G}$$

where V is the shearing force acting over an area A_s.

Poisson's Ratio

When a bar is subjected to a tensile loading there is an increase in length of the bar in the direction of the applied load, but there is also a decrease in a lateral dimension perpendicular to the load. The ratio of the sidewise deformation (or strain) to the longitudinal deformation (or strain) is called the Poisson's ratio and is denoted by ν. For most steel, it lies in the range of 0.25 to 0.3, and 0.20 for concrete.



$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

where ε_x is strain in the x-direction and ε_y and ε_z are the strains in the perpendicular direction. The negative sign indicates a decrease in the transverse dimension when ε_x is positive.

BIAXIAL DEFORMATION

If an element is subjected simultaneously by ensile stresses, σ_x and σ_y, in the x and y directions, the strain in the x-direction is σ_x / E and the strain in the y direction is σ_y / E. Simultaneously, the stress in the y direction will produce a lateral contraction on the x direction of the amount $-\nu\ \varepsilon_y$ or $-\nu\ \sigma_y/E$. The resulting strain in the x direction will be

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} \text{ or } \sigma_x = \frac{(\varepsilon_x + \nu\varepsilon_y)E}{1-\nu^2}$$

and

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} \text{ or } \sigma_y = \frac{(\varepsilon_y + \nu\varepsilon_x)E}{1-\nu^2}$$

TRIAXIAL DEFORMATION

If an element is subjected simultaneously by three mutually perpendicular normal stresses σ_x, σ_y, and σ_z, which are accompanied by strains ε_x, ε_y, and ε_z, respectively,

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$
$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$
$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

Tensile stresses and elongation are taken as positive. Compressive stresses and contraction are taken as negative.

Relationship Between E, G, and ν

The relationship between modulus of elasticity E, shear modulus G and Poisson's ratio ν is:

$$G = \frac{E}{2(1+\nu)}$$

Bulk Modulus of Elasticity or Modulus of Volume Expansion, K

The bulk modulus of elasticity K is a measure of a resistance of a material to change in volume without change in shape or form. It is given as

$$K = \frac{E}{3(1-2\nu)} = \frac{\sigma}{\Delta V / V}$$

where V is the volume and ∆V is change in volume. The ratio ∆V / V is called volumetric strain and can be expressed as

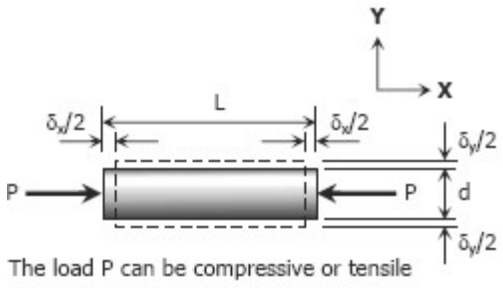
$$\frac{\Delta V}{V} = \frac{\sigma}{K} = \frac{3(1-2\nu)}{E}$$

Solved Problems in Shearing Deformation

Problem 222

A solid cylinder of diameter d carries an axial load P. Show that its change in diameter is 4Pν / πEd.

Solution 222



$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$

$$\varepsilon_y = -\nu\varepsilon_x$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E}$$

$$\frac{\delta_y}{d} = -\nu\frac{-P}{AE}$$

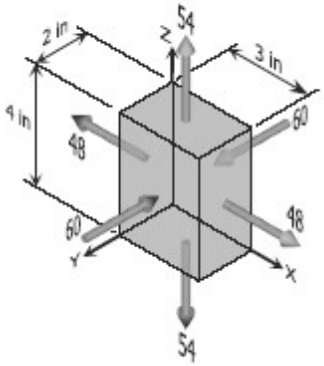
$$\delta_y = \nu\frac{Pd}{\frac{1}{4}\pi d^2 E}$$

$$\delta_y = \frac{4P\nu}{\pi E d} \quad ok!$$

Problem 223

A rectangular steel block is 3 inches long in the x direction, 2 inches long in the y direction, and 4 inches long in the z direction. The block is subjected to a triaxial loading of three uniformly distributed forces as follows: 48 kips tension in the x direction, 60 kips compression in the y direction, and 54 kips tension in the z direction. If $\nu = 0.30$ and $E = 29 \times 10^6$ psi, determine the single uniformly distributed load in the x direction that would produce the same deformation in the y direction as the original loading.

Solution 223



For triaxial deformation (tensile triaxial stresses):
(compressive stresses are negative stresses)

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\sigma_x = \frac{P_x}{A_{yz}} = \frac{48}{4(2)} = 6.0 \text{ ksi (tension)}$$

$$\sigma_y = \frac{P_y}{A_{xz}} = \frac{60}{4(3)} = 5.0 \text{ ksi (compression)}$$

$$\sigma_z = \frac{P_z}{A_{xy}} = \frac{54}{2(3)} = 9.0 \text{ ksi (tension)}$$

$$\varepsilon_y = \frac{1}{29\times10^6}[-5000 - 0.30(6000 + 9000)]$$

$$\varepsilon_y = -3.276 \times 10^{-4}$$

ε_y is negative, thus tensile force is required in the x-direction to produce the same deformation in the y-direction as the original forces.

For equivalent single force in the x-direction:
(uniaxial stress)

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$

$$-\nu\varepsilon_x = \varepsilon_y$$

$$-\nu\frac{\sigma_x}{E} = \varepsilon_y$$

$$-0.30\left(\frac{\sigma_x}{29\times10^6}\right) = -3.276 \times 10^{-4}$$

$$\sigma_x = 31\,666.67 \text{ psi}$$

$$\sigma_x = \frac{P_x}{4(2)} = 31\,666.67$$

$$P_x = 253\,333.33 \text{ lb (tension)}$$

$$P_x = \mathbf{253.33 \text{ kips (tension)}}$$

Problem 224

For the block loaded triaxially as described in Prob. 223, find the uniformly distributed load that must be added in the x direction to produce no deformation in the z direction.

Solution 224

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

$\sigma_x = 6.0$ ksi (tension)
$\sigma_y = 5.0$ ksi (compression)
$\sigma_z = 9.0$ ksi (tension)

$$\varepsilon_z = \frac{1}{29\times10^6}[9000 - 0.3(6000 - 5000)]$$

$$\varepsilon_z = 2.07 \times 10^{-5}$$

ε_z is positive, thus positive stress is needed in the x-direction to eliminate deformation in z-direction.

The application of loads is still simultaneous:
(No deformation means zero strain)

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] = 0$$

$\sigma_z = \nu(\sigma_x + \sigma_y)$
$\sigma_y = 5.0$ ksi → (compression)
$\sigma_z = 9.0$ ksi → (tension)

$9000 = 0.30(\sigma_x - 5000)$

$\sigma_x = 35\,000$ psi

$\sigma_{added} + 6000 = 35\,000$

$\sigma_{added} = 29\,000$ psi

$$\frac{P_{added}}{2(4)} = 29\,000$$

$P_{added} = 232\,000$ lb

$P_{added} = \mathbf{232\ kips}$

Problem 225

A welded steel cylindrical drum made of a 10-mm plate has an internal diameter of 1.20 m. Compute the change in diameter that would be caused by an internal pressure of 1.5 MPa. Assume that Poisson's ratio is 0.30 and E = 200 GPa.

Solution 225

σ_y = longitudinal stress

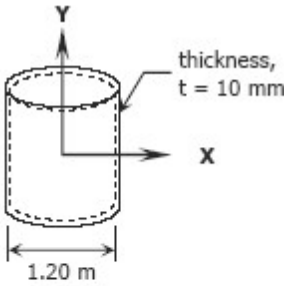
$$\sigma_y = \frac{pD}{4t} = \frac{1.5(1200)}{4(10)}$$

$$\sigma_y = 45 \text{ MPa}$$

σ_x = tangential stress

$$\sigma_x = \frac{pD}{2t} = \frac{1.5(1200)}{2(10)}$$

$$\sigma_x = 90 \text{ MPa}$$



$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$

$$\varepsilon_x = \frac{90}{200000} - 0.3\left(\frac{45}{200000}\right)$$

$$\varepsilon_x = 3.825 \times 10^{-4}$$

$$\varepsilon_x = \frac{\Delta D}{D}$$

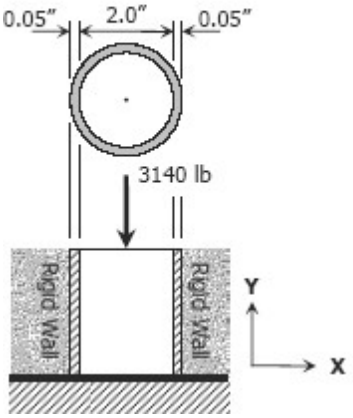
$$\Delta D = \varepsilon_x D = (3.825 \times 10^{-4})(1200)$$

$$\Delta D = \mathbf{0.459 \text{ mm}}$$

Problem 226

A 2-in.-diameter steel tube with a wall thickness of 0.05 inch just fits in a rigid hole. Find the tangential stress if an axial compressive load of 3140 lb is applied. Assume ν = 0.30 and neglect the possibility of buckling.

Solution 226



$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = 0$$

$$\sigma_x = \nu\sigma_y$$

where σ_x = tangential stress

σ_y = longitudinal stress

$$\sigma_y = \frac{P_y}{A} = \frac{3140}{\pi(2)(0.05)}$$

$$\sigma_y = 31400/\pi \text{ psi}$$

$$\sigma_x = 0.30(31400/\pi)$$

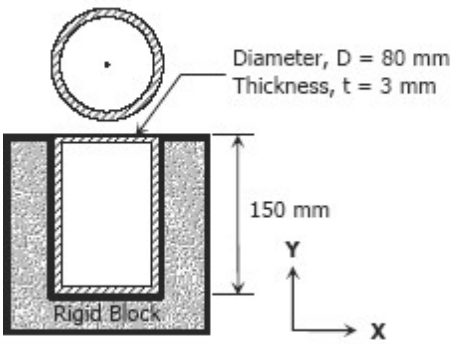
$$\sigma_x = 9430/\pi \text{ psi}$$

$$\sigma_x = \mathbf{2298.5 \text{ psi}}$$

Problem 227

A 150-mm-long bronze tube, closed at its ends, is 80 mm in diameter and has a wall thickness of 3 mm. It fits without clearance in an 80-mm hole in a rigid block. The tube is then subjected to an internal pressure of 4.00 MPa. Assuming $\nu = 1/3$ and E = 83 GPa, determine the tangential stress in the tube.

Solution 227



Longitudinal stress:

$$\sigma_y = \frac{pD}{4t} = \frac{4(80)}{4(3)}$$

$$\sigma_y = \frac{80}{3} \text{ MPa}$$

The strain in the x-direction is:

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = 0$$

$$\sigma_x = \nu\sigma_y = \text{tangential stress}$$

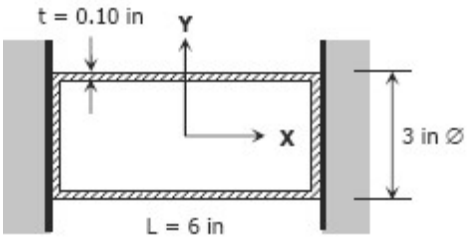
$$\sigma_x = \frac{1}{3}\left(\frac{80}{3}\right)$$

$$\sigma_x = 8.89 \text{ MPa}$$

Problem 228

A 6-in.-long bronze tube, with closed ends, is 3 in. in diameter with a wall thickness of 0.10 in. With no internal pressure, the tube just fits between two rigid end walls. Calculate the longitudinal and tangential stresses for an internal pressure of 6000 psi. Assume $\nu = 1/3$ and $E = 12 \times 10^6$ psi.

Solution 228



$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = 0$$

$\sigma_x = \nu\sigma_y = \sigma_l \rightarrow$ longitudinal stress

$\sigma_t = \sigma_y \rightarrow$ tangential stress

$$\sigma_t = \frac{pD}{2t} = \frac{6000(3)}{2(0.10)}$$

$$\sigma_t = 90{,}000 \text{ psi}$$

$$\sigma_l = \nu\sigma_y = \tfrac{1}{3}(90{,}000)$$

$$\sigma_l = 30{,}000 \text{ psi}$$

Statically Indeterminate Members

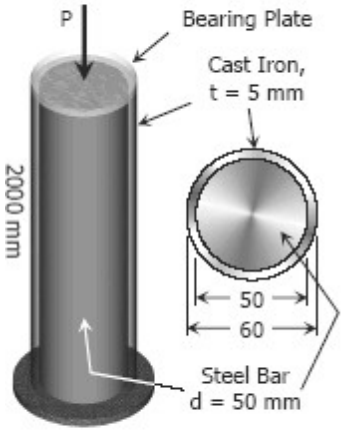
When the reactive forces or the internal resisting forces over a cross section exceed the number of independent equations of equilibrium, the structure is called **statically indeterminate**. These cases require the use of additional relations that depend on the elastic deformations in the members.

Solved Problems in Statically Indeterminate Members

Problem 233

A steel bar 50 mm in diameter and 2 m long is surrounded by a shell of a cast iron 5 mm thick. Compute the load that will compress the combined bar a total of 0.8 mm in the length of 2 m. For steel, E = 200 GPa, and for cast iron, E = 100 GPa.

Solution 233



$$\delta = \frac{PL}{AE}$$

$$\delta = \delta_{cast\ iron} = \delta_{steel} = 0.8\ \text{mm}$$

$$\delta_{cast\ iron} = \frac{P_{cast\ iron}(2000)}{[\frac{1}{4}\pi(60^2 - 50^2)](100\,000)} = 0.8$$

$$P_{cast\ iron} = 11\,000\pi\ \text{N}$$

$$\delta_{steel} = \frac{P_{steel}(2000)}{[\frac{1}{4}\pi(50^2)](200\,000)} = 0.8$$

$$P_{steel} = 50\,000\pi\ \text{N}$$

$$\sum F_V = 0$$

$$P = P_{cast\ iron} + P_{steel}$$

$$P = 11\,000\pi + 50\,000\pi$$

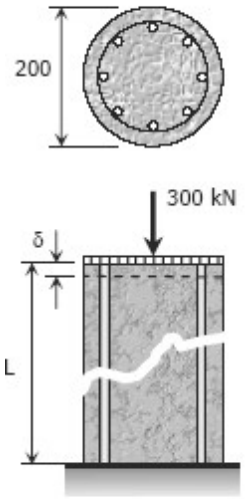
$$P = 61\,000\pi\ \text{N}$$

$$P = \mathbf{191.64\ kN}$$

Problem 234

A reinforced concrete column 200 mm in diameter is designed to carry an axial compressive load of 300 kN. Determine the required area of the reinforcing steel if the allowable stresses are 6 MPa and 120 MPa for the concrete and steel, respectively. Use E_{co} = 14 GPa and E_{st} = 200 GPa.

Solution 234



$$\delta_{co} = \delta_{st} = \delta$$

$$\left(\frac{PL}{AE}\right)_{co} = \left(\frac{PL}{AE}\right)_{st}$$

$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$

$$\frac{\sigma_{co} L}{14\,000} = \frac{\sigma_{st} L}{200\,000}$$

$$100\sigma_{co} = 7\sigma_s$$

When σ_{st} = 120 MPa

$100\sigma_{co} = 7(120)$

σ_{co} = 8.4 MPa > 6 MPa (not ok!)

When σ_{co} = 6 MPa

$100(6) = 7\sigma_{st}$

σ_{st} = 85.71 MPa < 120 MPa (ok!)

Use σ_{co} = 6 MPa and σ_{st} = 85.71 MPa

$$\sum F_V = 0$$

$$P_{st} + P_{co} = 300$$

$$\sigma_{st} A_{st} + \sigma_{co} A_{co} = 300$$

$$85.71A_{st} + 6[\tfrac{1}{4}\pi(200)^2 - A_{st}] = 300(1000)$$

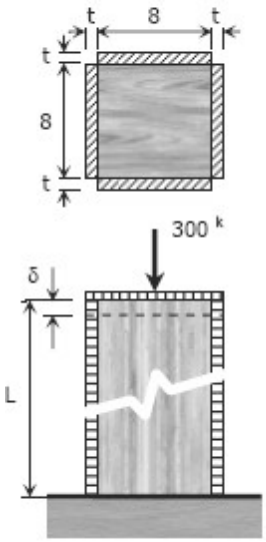
$$79.71A_{st} + 60\,000\pi = 300\,000$$

$$A_{st} = \mathbf{1398.9\ mm^2}$$

Problem 235

A timber column, 8 in. × 8 in. in cross section, is reinforced on each side by a steel plate 8 in. wide and t in. thick. Determine the thickness t so that the column will support an axial load of 300 kips without exceeding a maximum timber stress of 1200 psi or a maximum steel stress of 20 ksi. The moduli of elasticity are 1.5 × 10^6 psi for timber, and 29 × 10^6 psi for steel.

Solution 235



$\delta_{st} = \delta_t$

$$\left(\frac{\sigma L}{E}\right)_{st} = \left(\frac{\sigma L}{E}\right)_{timber}$$

$$\frac{\sigma_{st} L}{29\times10^6} = \frac{\sigma_{timber} L}{1.5\times10^6}$$

$1.5\sigma_{st} = 29\sigma_{timber}$

When $\sigma_{timber} = 1200$ psi

$1.5\sigma_{st} = 29(1200)$

$\sigma_{st} = 23\,200$ psi $= 23.2$ ksi > 20 ksi (not ok!)

When $\sigma_{st} = 20$ ksi

$1.5(20 \times 1000) = 29\sigma_{timber}$

$\sigma_{timber} = 1034.48$ psi < 1200 psi (ok!)

Use $\sigma_{st} = 20$ ksi and $\sigma_{timber} = 1.03$ ksi

$\sum F_V = 0$

$F_{steel} + F_{timber} = 300$

$\sigma_{st} A_{st} + \sigma_{timber} A_{timber} = 300$

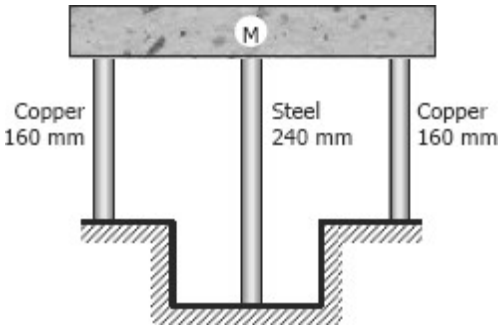
$20[4(8t)] + 1.03(8^2) = 300$

$t = \mathbf{0.365\ in}$

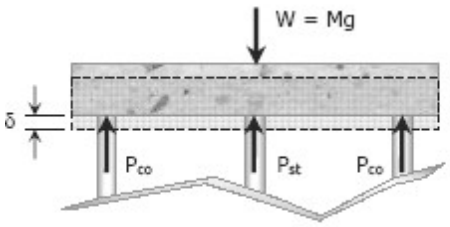
Problem 236

A rigid block of mass M is supported by three symmetrically spaced rods as shown in fig P-236. Each copper rod has an area of 900 mm^2; E = 120 GPa; and the allowable stress is 70 MPa. The steel rod has an area of 1200 mm^2; E = 200 GPa; and the allowable stress is 140 MPa. Determine the largest mass M which can be supported.

Figure P-236 and P-237



Solution 236



$$\delta_{co} = \delta_{st}$$

$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$

$$\frac{\sigma_{co}(160)}{120000} = \frac{\sigma_{st}(240)}{200000}$$

$$10\sigma_{co} = 9\sigma_{st}$$

When $\sigma_{st} = 140$ MPa

$\sigma_{co} = \frac{9}{10}(140)$

$\sigma_{co} = 126$ MPa > 70 MPa (not ok!)

When $\sigma_{co} = 70$ MPa

$\sigma_{st} = \frac{10}{9}(70)$

$\sigma_{st} = 77.78$ MPa < 140 MPa (ok!)

Use $\sigma_{co} = 70$ MPa and $\sigma_{st} = 77.78$ MPa

$\sum F_V = 0$

$2P_{co} + P_{st} = W$

$2(\sigma_{co}A_{co}) + \sigma_{st}A_{st} = Mg$

$2[70(900)] + 77.78(1200) = M(9.81)$

$M = \mathbf{22\,358.4\ kg}$

Problem 237

In Prob. 236, how should the lengths of the two identical copper rods be changed so that each material will be stressed to its allowable limit?

Solution 237

Use $\sigma_{co} = 70$ MPa and $\sigma_{st} = 140$ MPa

$\delta_{co} = \delta_{st}$

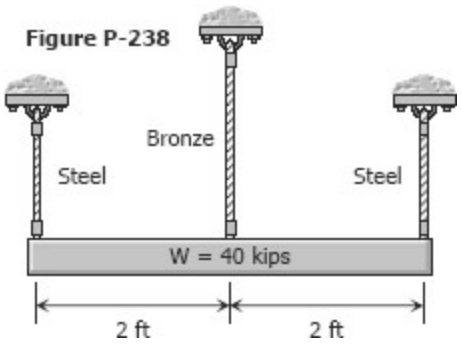
$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$

$$\frac{70L_{co}}{120000} = \frac{140(240)}{200000}$$

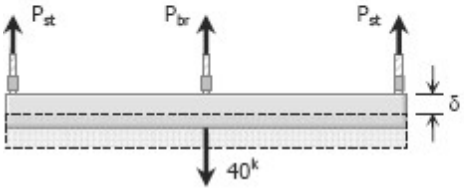
$L_{co} = \mathbf{288\ mm}$

Problem 238

The lower ends of the three bars in Fig. P-238 are at the same level before the uniform rigid block weighing 40 kips is attached. Each steel bar has a length of 3 ft, and area of 1.0 in.^{2}, and E = 29 × 10^6 psi. For the bronze bar, the area is 1.5 in.^{2} and E = 12 × 10^6 psi. Determine (a) the length of the bronze bar so that the load on each steel bar is twice the load on the bronze bar, and (b) the length of the bronze that will make the steel stress twice the bronze stress.



Solution 238



(a) Condition: $P_{st} = 2P_{br}$

$\sum F_V = 0$

$2P_{st} + P_{br} = 40$

$2(2P_{br}) + P_{br} = 40$

$P_{br} = 8$ kips

$P_{st} = 2(8) = 16$ kips

$\delta_{br} = \delta_{st}$

$$\left(\frac{PL}{AE}\right)_{br} = \left(\frac{PL}{AE}\right)_{st}$$

$$\frac{8000L_{br}}{1.5(12\times10^6)} = \frac{16000(3\times12)}{1.0(29\times10^6)}$$

$L_{br} = 44.69$ in

$L_{br} = 3.72$ ft

(b) Condition: $\sigma_{st} = 2\sigma_{br}$

$\sum F_V = 0$

$2P_{st} + P_{br} = 40$

$2(\sigma_{st}A_{st}) + \sigma_{br}A_{br} = 40$

$2[(2\sigma_{br})A_{st}] + \sigma_{br}A_{br} = 40$

$4\sigma_{br}(1.0) + \sigma_{br}(1.5) = 40$

$\sigma_{br} = 7.27$ ksi

$\sigma_{st} = 2(7.27) = 14.54$ ksi

$\delta_{br} = \delta_{st}$

$$\left(\frac{\sigma L}{E}\right)_{br} = \left(\frac{\sigma L}{E}\right)_{st}$$

$$\frac{7.27(1000)L_{br}}{12\times10^6} = \frac{14.54(1000)(3\times12)}{29\times10^6}$$

$L_{br} = 29.79$ in

$L_{br} = 2.48$ ft

Problem 239

The rigid platform in Fig. P-239 has negligible mass and rests on two steel bars, each 250.00 mm long. The center bar is aluminum and 249.90 mm long. Compute the stress in the aluminum bar after the center load P = 400 kN has been applied. For each steel bar, the area is 1200 mm^2 and E = 200 GPa. For the aluminum bar, the area is 2400 mm^2 and E = 70 GPa.

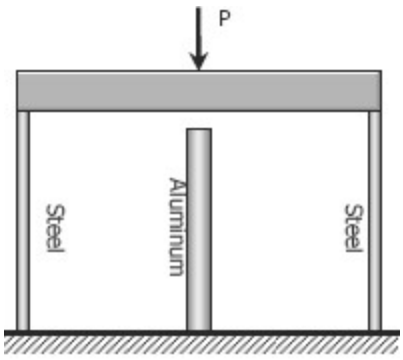
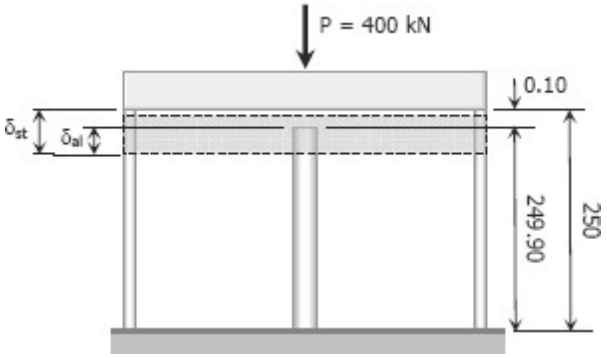


Figure P-239

Solution 239



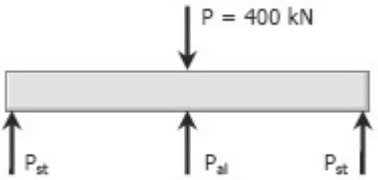
$$\delta_{st} = \delta_{al} + 0.10$$

$$\left(\frac{\sigma L}{E}\right)_{st} = \left(\frac{\sigma L}{E}\right)_{al} + 0.10$$

$$\frac{\sigma_{st}(250)}{200\,000} = \frac{\sigma_{al}(249.90)}{70\,000} + 0.10$$

$$0.00125\sigma_{st} = 0.00357\sigma_{al} + 0.10$$

$$\sigma_{st} = 2.856\sigma_{al} + 80$$



$$\sum F_V = 0$$

$$2P_{st} + P_{al} = 400\,000$$

$$2\sigma_{st} A_{st} + \sigma_{al} A_{al} = 400\,000$$

$$2(2.856\sigma_{al} + 80)1200 + \sigma_{al}(2400) = 400\,000$$

$$9254.4\sigma_{al} + 192\,000 = 400\,000$$

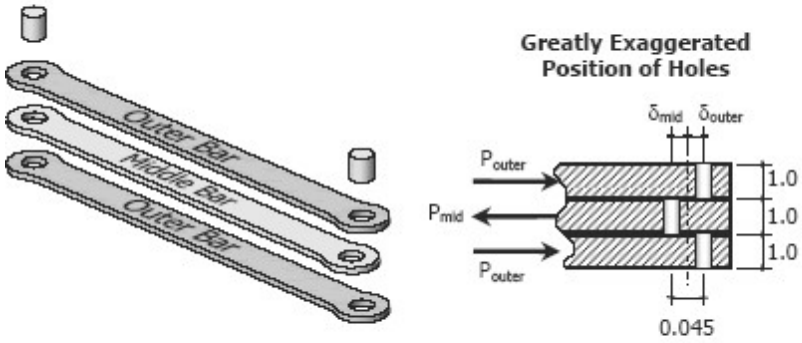
$$\sigma_{al} = \mathbf{22.48\ MPa}$$

Problem 240

Three steel eye-bars, each 4 in. by 1 in. in section, are to be assembled by driving rigid 7/8-in.-diameter drift pins through holes drilled in the ends of the bars. The center-line spacing between the holes is 30 ft in the two outer bars, but 0.045 in. shorter in the middle bar. Find the shearing stress developed in the drip pins. Neglect local deformation at the holes.

Solution 240

Middle bar is 0.045 inch shorter between holes than outer bars.



$$\sum F_H = 0$$
$$P_{mid} = 2P_{outer}$$

$$\delta_{outer} + \delta_{mid} = 0.045$$

$$\left(\frac{PL}{AE}\right)_{outer} + \left(\frac{PL}{AE}\right)_{mid} = 0.045$$

$$\frac{P_{outer}(30\times12)}{[1.0(4.0)]E} + \frac{P_{mid}(30\times12-0.045)}{[1.0(4.0)]E} = 0.045$$

$$360P_{outer} + 359.955P_{mid} = 0.18E$$
$$360P_{outer} + 359.955(2P_{outer}) = 0.18E$$

(For steel: E = 29 × 106 psi)

$$1079.91P_{outer} = 0.18(29\times10^6)$$
$$P_{outer} = 4833.74 \text{ lb}$$

$$P_{mid} = 2(4833.74)$$
$$P_{mid} = 9667.48 \text{ lb}$$

Use shear force $V = P_{mid}$

Shearing stress of drip pins (double shear):

$$\tau = \frac{V}{A} = \frac{9667.48}{2[\frac{1}{4}\pi(\frac{7}{8})^2]}$$

$$\tau = \mathbf{8038.54 \text{ psi}}$$

Problem 241

As shown in Fig. P-241, three steel wires, each 0.05 in.^{2} in area, are used to lift a load W = 1500 lb. Their unstressed lengths are 74.98 ft, 74.99 ft, and 75.00 ft. (a) What stress exists in the longest wire? (b) Determine the stress in the shortest wire if W = 500 lb.

Solution 241

Let $L_1 = 74.98$ ft; $L_2 = 74.99$ ft; and $L_3 = 75.00$ ft

(a) Bring L_1 and L_2 into $L_3 = 75$ ft length:
(For steel: E = 29 × 106 psi)

$$\delta = \frac{PL}{AE}$$

For L_1:

$$(75 - 74.98)(12) = \frac{P_1(74.98 \times 12)}{0.05(29 \times 10^6)}$$

$$P_1 = 386.77 \text{ lb}$$

For L_2

$$(75 - 74.99)(12) = \frac{P_2(74.99 \times 12)}{0.05(29 \times 10^6)}$$

$$P_2 = 193.36 \text{ lb}$$

Figure P-241

Let $P = P_3$ (Load carried by L_3)
$P + P_2$ (Total load carried by L_2)
$P + P_1$ (Total load carried by L_1)

$\sum F_V = 0$
$(P + P_1) + (P + P_2) + P = W$
$3P + 386.77 + 193.36 = 1500$
$P = 306.62$ lb $= P_3$

$$\sigma_3 = \frac{P_3}{A} = \frac{306.62}{0.05}$$

$\sigma_3 =$ **6132.47 psi**

(b) From the above solution:
$P_1 + P_2 = 580.13$ lb > 500 lb (L_3 carries no load)

Bring L_1 into $L_2 = 74.99$ ft

$$\left[\delta = \frac{PL}{AE}\right] \quad (74.99 - 74.98)(12) = \frac{P_1(74.98 \times 12)}{0.05(29 \times 10^6)}$$

$$P_1 = 193.38 \text{ lb}$$

Let $P = P_2$ (Load carried by L_2)
$P + P_1$ (Total load carried by L_1)

$\sum F_V = 0$
$(P + P_1) + P = 500$
$2P + 193.38 = 500$
$P = 153.31$ lb

$P + P_1 = 153.31 + 193.38$
$P + P_1 = 346.69$ lb

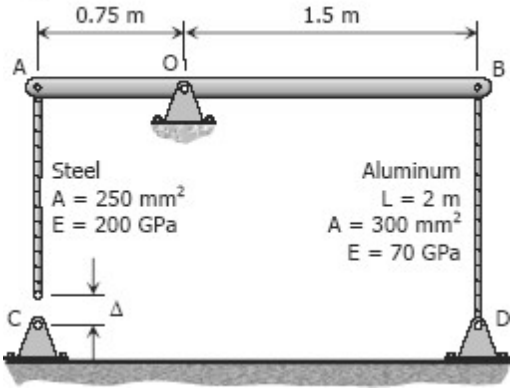
$$\sigma = \frac{P + P_1}{A} = \frac{346.69}{0.05}$$

$\sigma =$ **6933.8 psi**

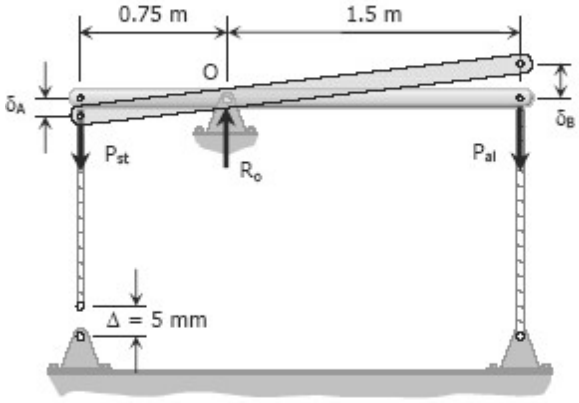
Problem 242

The assembly in Fig. P-242 consists of a light rigid bar AB, pinned at O, that is attached to the steel and aluminum rods. In the position shown, bar AB is horizontal and there is a gap, Δ = 5 mm, between the lower end of the steel rod and its pin support at C. Compute the stress in the aluminum rod when the lower end of the steel rod is attached to its support.

Figure P-242



Solution 242



$\sum M_O = 0$

$0.75P_{st} = 1.5P_{al}$

$P_{st} = 2P_{al}$

$\sigma_{st} A_{st} = 2(\sigma_{al} A_{al})$

$$\sigma_{st} = \frac{2(\sigma_{al} A_{al})}{A_{st}}$$

$$\sigma_{st} = \frac{2[\sigma_{al}(300)]}{250}$$

$\sigma_{st} = 2.4\sigma_{al}$

$\delta_{al} = \delta_B$

By ratio and proportion:

$$\frac{\delta_A}{0.75} = \frac{\delta_B}{1.5}$$

$\delta_A = 0.5\delta_B$

$\delta_A = 0.5\delta_{al}$

$\Delta = \delta_{st} + \delta_A$

$5 = \delta_{st} + 0.5\delta_{al}$

$$5 = \frac{\sigma_{st}(2000 - 5)}{250(200\,000)} + 0.5\left[\frac{\sigma_{al}(2000)}{300(70000)}\right]$$

$5 = (3.99 \times 10^{-5})\,\sigma_{st} + (4.76 \times 10^{-5})\,\sigma_{al}$

$\sigma_{al} = 105\,000 - 0.8379\sigma_{st}$

$\sigma_{al} = 105\,000 - 0.8379(2.4\sigma_{al})$

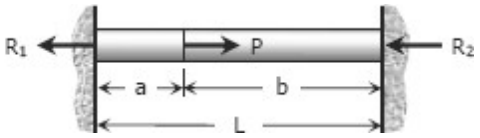
$3.01096\sigma_{al} = 105\,000$

$\sigma_{al} = \mathbf{34\,872.6\ MPa}$

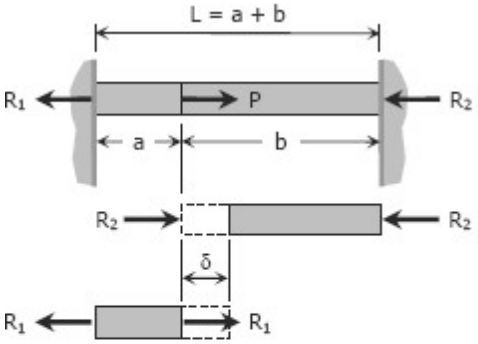
Problem 243

A homogeneous rod of constant cross section is attached to unyielding supports. It carries an axial load P applied as shown in Fig. P-243. Prove that the reactions are given by R_1 = Pb/L and R_2 = Pa/L.

Figure P-243



Solution 243



$$\sum F_H = 0$$
$$R_1 + R_2 = P$$
$$R_2 = P - R_1$$

$$\delta_1 = \delta_2 = \delta$$
$$\left(\frac{PL}{AE}\right)_1 = \left(\frac{PL}{AE}\right)_2$$
$$\frac{R_1 a}{AE} = \frac{R_2 b}{AE}$$
$$R_1 a = R_2 b$$

$$R_1 a = (P - R_1) b$$
$$R_1 a = Pb - R_1 b$$
$$R_1 (a + b) = Pb$$
$$R_1 L = Pb$$
$$R_1 = Pb/L \qquad \text{ok!}$$

$$R_2 = P - Pb/L$$
$$R_2 = \frac{P(L-b)}{L}$$
$$R_2 = Pa/L \qquad ok!$$

Problem 244

A homogeneous bar with a cross sectional area of 500 mm^2 is attached to rigid supports. It carries the axial loads P1 = 25 kN and P2 = 50 kN, applied as shown in Fig. P-244. Determine the stress in segment BC. (Hint: Use the results of Prob. 243, and compute the reactions caused by P_1 and P_2 acting separately. Then use the principle of superposition to compute the reactions when both loads are applied.)

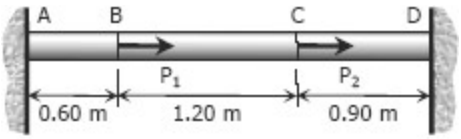
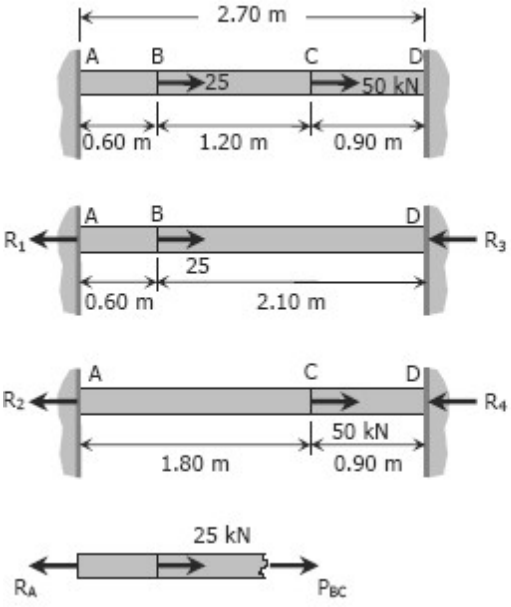


Figure P-244

Solution 244



From the result of Prob. 243:

$R_1 = 25(2.10)/2.70$

$R_1 = 19.44$ kN

$R_2 = 50(0.90)/2.70$

$R_2 = 16.67$ kN

$R_A = R_1 + R_2$

$R_A = 19.44 + 16.67$

$R_A = 36.11$ kN

For segment BC

$P_{BC} + 25 = R_A$

$P_{BC} + 25 = 36.11$

$P_{BC} = 11.11$ kN

$$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{11.11(1000)}{500}$$

$\sigma_{BC} = 22.22$ MPa

Problem 245

The composite bar in Fig. P-245 is firmly attached to unyielding supports. Compute the stress in each material caused by the application of the axial load P = 50 kips.

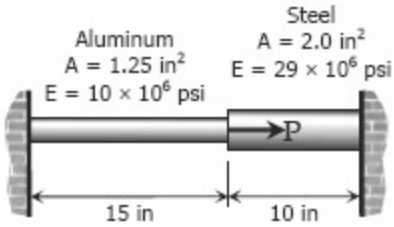
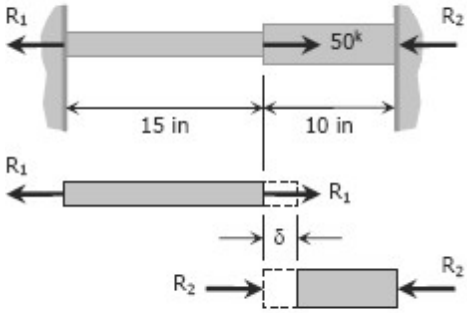


Figure P-245 and P-246

Solution 245

$$\sum F_H = 0$$
$$R_1 + R_2 = 50\,000$$
$$R_1 = 50\,000 - R_2$$



$$\delta_{al} = \delta_{st} = \delta$$
$$\left(\frac{PL}{AE}\right)_{al} = \left(\frac{PL}{AE}\right)_{st}$$
$$\frac{R_1(15)}{1.25(10\times10^6)} = \frac{R_2(10)}{2.0(29\times10^6)}$$
$$R_2 = 6.96R_1$$
$$R_2 = 6.96(50\,000 - R_2)$$
$$7.96R_2 = 348\,000$$
$$R_2 = 43\,718.59 \text{ lb}$$

$$\sigma_{st} = \frac{R_2}{A_{st}} = \frac{43718.59}{2.0}$$
$$\sigma_{st} = \mathbf{21\,859.30 \text{ psi}}$$

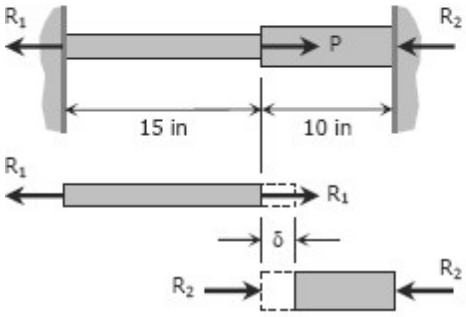
$$R_1 = 50\,000 - 43\,718.59$$
$$R_1 = 6281.41 \text{ lb}$$

$$\sigma_{al} = \frac{R_1}{A_{al}} = \frac{6281.41}{1.25}$$
$$\sigma_{al} = \mathbf{5025.12 \text{ psi}}$$

Problem 246

Referring to the composite bar in Prob. 245, what maximum axial load P can be applied if the allowable stresses are 10 ksi for aluminum and 18 ksi for steel.

Solution 246



$\delta_{st} = \delta_{al} = \delta$

$$\left(\frac{\sigma L}{E}\right)_{st} = \left(\frac{\sigma L}{E}\right)_{al}$$

$$\frac{\sigma_{st}(10)}{29\times10^6} = \frac{\sigma_{al}(15)}{10\times10^6}$$

$\sigma_{st} = 4.35\sigma_{al}$

When $\sigma_{al} = 10$ ksi

$\sigma_{st} = 4.35(10)$

$\sigma_{st} = 43.5$ ksi > 18 ksi (not ok!)

When $\sigma_{st} = 18$ ksi

$18 = 4.35\sigma_{al}$

$\sigma_{al} = 4.14$ ksi < 10 ksi (ok!)

Use $\sigma_{al} = 4.14$ ksi and $\sigma_{st} = 18$ ksi

$\sum F_H = 0$

$P = R_1 + R_2$

$P = \sigma_{al} A_{al} + \sigma_{st} A_{st}$

$P = 4.14(1.25) + 18(2.0)$

$P = \mathbf{41.17\ kips}$

Problem 247

The composite bar in Fig. P-247 is stress-free before the axial loads P1 and P2 are applied. Assuming that the walls are rigid, calculate the stress in each material if P_1 = 150 kN and P_2 = 90 kN.

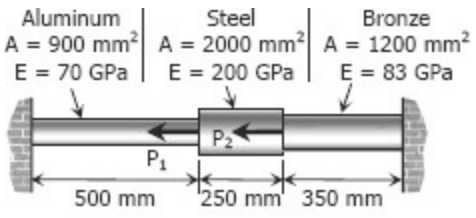
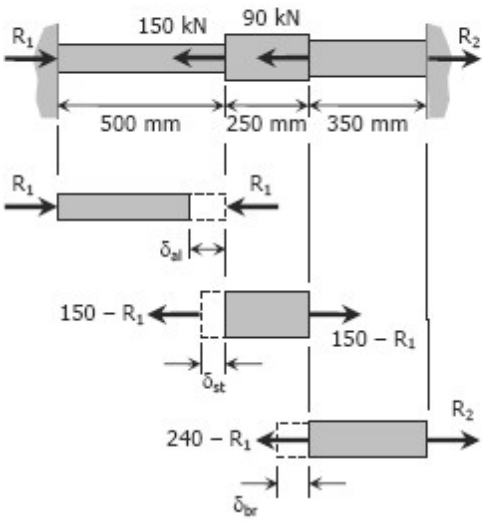


Figure P-247 and P-248

Solution 247



From the *FBD* of each material shown:

δ_{al} is shortening

δ_{st} and δ_{br} are lengthening

$R_2 = 240 - R_1$

$P_{al} = R_1$

$P_{st} = 150 - R_1$

$P_{br} = R_2 = 240 - R_1$

$$\delta_{al} = \delta_{st} + \delta_{br}$$

$$\left(\frac{PL}{AE}\right)_{al} = \left(\frac{PL}{AE}\right)_{st} + \left(\frac{PL}{AE}\right)_{br}$$

$$\frac{R_1(500)}{900(70000)} = \frac{(150-R_1)(250)}{2000(200000)} + \frac{(240-R_1)(350)}{1200(83000)}$$

$$\frac{R_1}{126000} = \frac{150-R_1}{1600000} + \frac{(240-R_1)7}{1992000}$$

$$\tfrac{1}{63}R_1 = \tfrac{1}{800}(150 - R_1) + \tfrac{7}{996}(240 - R_1)$$

$$(\tfrac{1}{63} + \tfrac{1}{800} + \tfrac{7}{996})R_1 = \tfrac{1}{800}(150) + \tfrac{7}{996}(240)$$

$R_1 = 77.60$ kN

$P_{al} = R_1 = 77.60$ kN

$P_{st} = 150 - 77.60 = 72.40$ kN

$P_{br} = 240 - 77.60 = 162.40$ kN

$\sigma = P/A$

$\sigma_{al} = 77.60(1000)/900$

$= \mathbf{86.22\ MPa}$

$\sigma_{st} = 72.40(1000)/2000$

$= \mathbf{36.20\ MPa}$

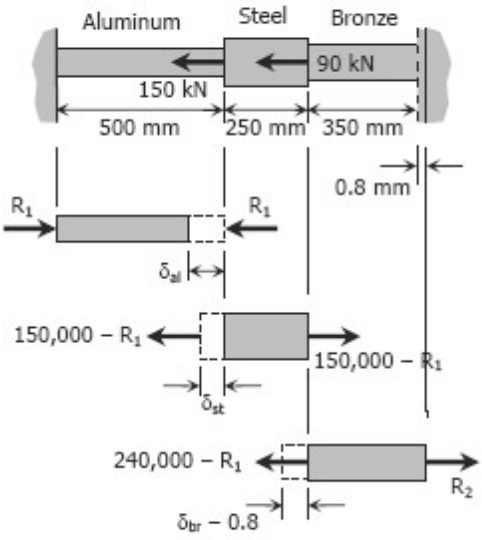
$\sigma_{br} = 162.40(1000)/1200$

$= \mathbf{135.33\ MPa}$

Problem 248

Solve Prob. 247 if the right wall yields 0.80 mm.

Solution 248



$$\delta_{al} = \delta_{st} + (\delta_{br} + 0.8)$$

$$\left(\frac{PL}{AE}\right)_{al} = \left(\frac{PL}{AE}\right)_{st} + \left(\frac{PL}{AE}\right)_{br} + 0.8$$

$$\frac{R_1(500)}{900(70000)} = \frac{(150000 - R_1)(250)}{2000(200000)} + \frac{(240000 - R_1)(350)}{1200(83000)} + 0.8$$

$$\frac{R_1}{126000} = \frac{150000 - R_1}{1600000} + \frac{7(240000 - R_1)}{1992000} + 0.8$$

$$\tfrac{1}{63} R_1 = \tfrac{1}{800}(150000 - R_1) + \tfrac{7}{996}(240000 - R_1) + 1600$$

$$(\tfrac{1}{63} + \tfrac{1}{800} + \tfrac{7}{996})R_1 = \tfrac{1}{800}(150000) + \tfrac{7}{996}(240000) + 1600$$

$$R_1 = 143\,854 \text{ N} = 143.854 \text{ kN}$$

$$P_{al} = R_1 = 143.854 \text{ kN}$$
$$P_{st} = 150 - R_1 = 150 - 143.854 = 6.146 \text{ kN}$$
$$P_{br} = R_2 = 240 - R_1 = 240 - 143.854 = 96.146 \text{ kN}$$

$$\sigma = P/A$$

$$\sigma_{al} = 143.854(1000)/900$$
$$= \mathbf{159.84 \text{ MPa}}$$

$$\sigma_{st} = 6.146(1000)/2000$$
$$= \mathbf{3.073 \text{ MPa}}$$

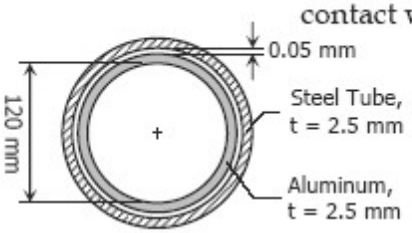
$$\sigma_{br} = 96.146(1000)/1200$$
$$= \mathbf{80.122 \text{ MPa}}$$

Problem 249

There is a radial clearance of 0.05 mm when a steel tube is placed over an aluminum tube. The inside diameter of the aluminum tube is 120 mm, and the wall thickness of each tube is 2.5 mm. Compute the contact pressure and tangential stress in each tube when the aluminum tube is subjected to an internal pressure of 5.0 MPa.

Solution 249

Internal pressure of aluminum tube to cause contact with the steel:



$$\delta_{al} = \left(\frac{\sigma L}{E}\right)_{al}$$

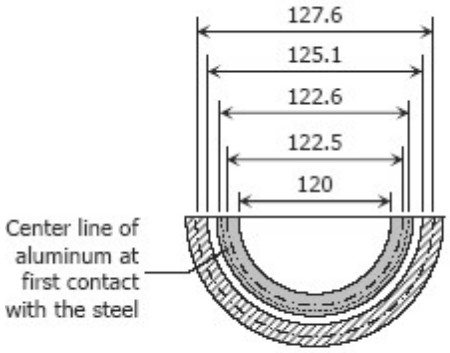
$$\pi(122.6 - 122.5) = \frac{\sigma_1(122.5\pi)}{70000}$$

$$\sigma_1 = 57.143 \text{ MPa}$$

$$\frac{p_1 D}{2t} = 57.143$$

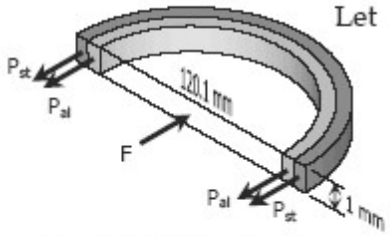
$$\frac{p_1(120)}{2(2.5)} = 57.143$$

$$p_1 = 2.381 \text{ MPa}$$



→ pressure that causes aluminum to contact with the steel, further increase of pressure will expand both aluminum and steel tubes.

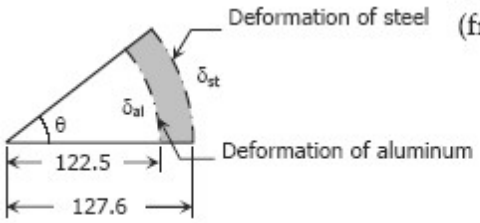
Let p_c = contact pressure between steel and aluminum tubes



FBD for p ≥ 2.381 MPa

$$2P_{st} + 2P_{al} = F$$
$$2P_{st} + 2P_{al} = 5.0(120.1)(1)$$
$$P_{st} + P_{al} = 300.25 \quad \rightarrow \text{Equation (1)}$$

The relationship of deformations is (from the figure):



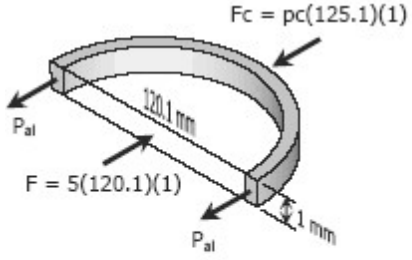
Geometric relation of deformations

$$\delta_{st} = 127.6\theta$$
$$\theta = \delta_{st} / 127.6$$

$$\delta_{al} = 122.5\theta$$
$$\delta_{al} = 122.5(\delta_{st} / 127.6)$$

$$\delta_{al} = 0.96\delta_{st}$$
$$\left(\frac{PL}{AE}\right)_{al} = 0.96\left(\frac{PL}{AE}\right)_{st}$$
$$\frac{P_{al}(122.5\pi)}{2.5(70000)} = 0.96\left[\frac{P_{st}(127.6\pi)}{2.5(200000)}\right]$$
$$P_{al} = 0.35P_{st} \quad \rightarrow \text{Equation (2)}$$

From Equation (1)



$$P_{st} + 0.35P_{st} = 300.25$$
$$P_{st} = 222.41 \text{ N}$$

$$P_{al} = 0.35(222.41)$$
$$P_{al} = 77.84 \text{ N}$$

Contact Force

$$F_c + 2P_{st} = F$$
$$p_c(125.1)(1) + 2(77.84) = 5(120.1)(1)$$
$$p_c = \mathbf{3.56 \text{ MPa}}$$

Problem 250

In the assembly of the bronze tube and steel bolt shown in Fig. P-250, the pitch of the bolt thread is p = 1/32 in.; the cross-sectional area of the bronze tube is 1.5 in.^{2} and of steel bolt is ¾ in.^{2} The nut is turned until there is a compressive stress of 4000 psi in the bronze tube. Find the stresses if the nut is given one additional turn. How many turns of the nut will reduce these stresses to zero? Use Ebr = 12 × 10^6 psi and Est = 29 × 10^6 psi.

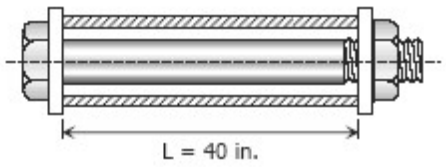


Figure P-250

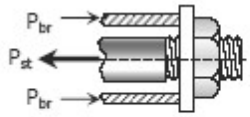
Solution 250

$P_{st} = P_{br}$

$A_{st}\,\sigma_{st} = P_{br}\,\sigma_{br}$

$\frac{3}{4}\,\sigma_{st} = 1.5\,\sigma_{br}$

$\sigma_{st} = 2\,\sigma_{br}$



For one turn of the nut:

$\delta_{st} + \delta_{br} = \frac{1}{32}$

$$\left(\frac{\sigma L}{E}\right)_{st} + \left(\frac{\sigma L}{E}\right)_{br} = \frac{1}{32}$$

$$\frac{\sigma_{st}(40)}{29\times10^6} + \frac{\sigma_{br}(40)}{12\times10^6} = \frac{1}{32}$$

$\sigma_{st} + \frac{29}{12}\,\sigma_{br} = 22\,656.25$

$2\sigma_{br} + \frac{29}{12}\,\sigma_{br} = 22\,656.25$

$\sigma_{br} = 5129.72$ psi

$\sigma_{st} = 2(5129.72) = 10\,259.43$ psi

Initial stresses:

$\sigma_{br} = 4000$ psi

$\sigma_{st} = 2(4000) = 8000$ psi

Final stresses:

$\sigma_{br} = 4000 + 5129.72 =$ **9129.72 psi**

$\sigma_{st} = 2(9129.72) =$ **18 259.4 psi**

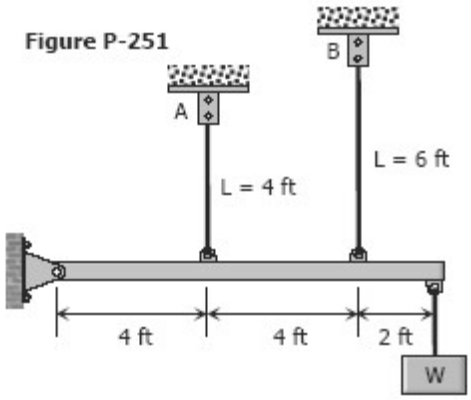
Required number of turns to reduce σ_{br} to zero:

$$n = \frac{9129.72}{5129.72} = \textbf{1.78 turns}$$

The nut must be turned back by 1.78 turns

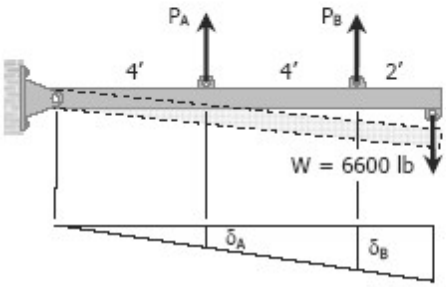
Problem 251

The two vertical rods attached to the light rigid bar in Fig. P-251 are identical except for length. Before the load W was attached, the bar was horizontal and the rods were stress-free. Determine the load in each rod if W = 6600 lb.



Solution 251

$$\sum M_{pin\ support} = 0$$
$$4P_A + 8P_B = 10(6600)$$
$$P_A + 2P_B = 16500 \qquad \rightarrow (1)$$



$$\frac{\delta_A}{4} = \frac{\delta_B}{8}$$
$$\delta_A = 0.5\delta_B$$
$$\left(\frac{PL}{AE}\right)_A = 0.5\left(\frac{PL}{AE}\right)_B$$
$$\frac{P_A(4)}{AE} = \frac{0.5P_B(6)}{AE}$$
$$P_A = 0.75P_B$$

From equation (1)
$$0.75P_B + 2P_B = 16500$$
$$P_B = \mathbf{6000\ lb}$$

$$P_A = 0.75(6000)$$
$$P_A = \mathbf{4500\ lb}$$

Problem 252

The light rigid bar ABCD shown in Fig. P-252 is pinned at B and connected to two vertical rods. Assuming that the bar was initially horizontal and the rods stress-free, determine the stress in each rod after the load after the load P = 20 kips is applied.

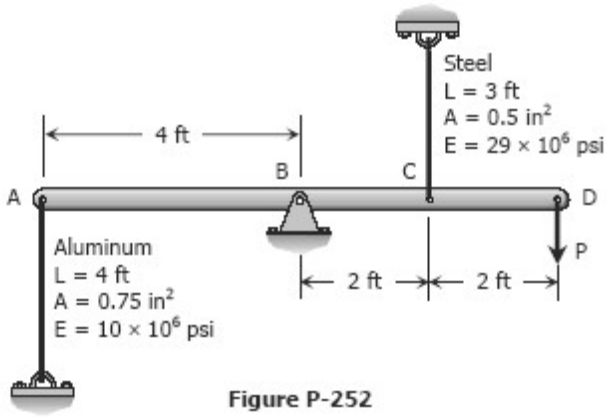
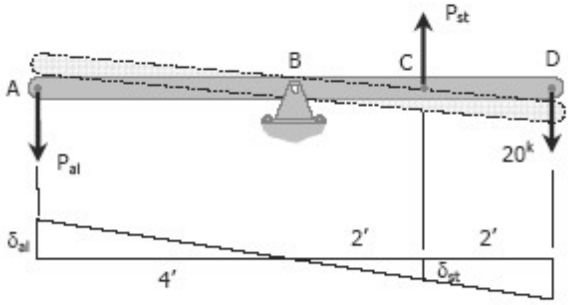


Figure P-252

Solution 252

$$\sum M_B = 0$$
$$4P_{al} + 2P_{st} = 4(20\,000)$$
$$4(\sigma_{al} A_{al}) + 2\sigma_{st} A_{st} = 80\,000$$
$$4[\sigma_{al}\,(0.75)] + 2[\sigma_{st}\,(0.5)] = 80\,000$$
$$3\sigma_{al} + \sigma_{st} = 80\,000 \quad \rightarrow (1)$$
$$\frac{\delta_{st}}{2} = \frac{\delta_{al}}{4}$$



$$\delta_{st} = 0.5\delta_{al}$$
$$\left(\frac{\sigma L}{E}\right)_{st} = 0.5\left(\frac{\sigma L}{E}\right)_{al}$$
$$\frac{\sigma_{st}(3)}{29\times10^6} = 0.5\left[\frac{P_{al}(4)}{10\times10^6}\right]$$
$$\sigma_{st} = \tfrac{29}{15}\,\sigma_{al}$$

From equation (1)

$$3\sigma_{al} + \tfrac{29}{15}\,\sigma_{al} = 80\,000$$
$$\sigma_{al} = 16\,216.22 \text{ psi}$$
$$\sigma_{al} = \mathbf{16.22 \text{ ksi}}$$

$$\sigma_{st} = \tfrac{29}{15}\,(16.22)$$
$$\sigma_{st} = \mathbf{31.35 \text{ ksi}}$$

Problem 253

As shown in Fig. P-253, a rigid beam with negligible weight is pinned at one end and attached to two vertical rods. The beam was initially horizontal before the load W = 50 kips was applied. Find the vertical movement of W.

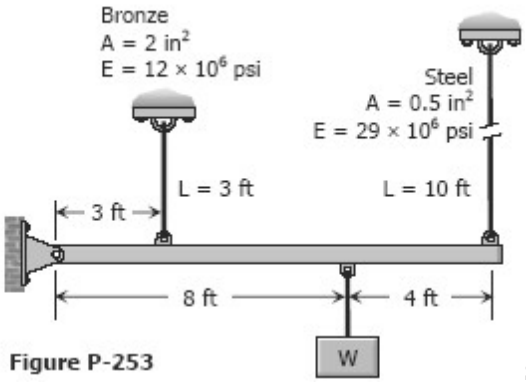
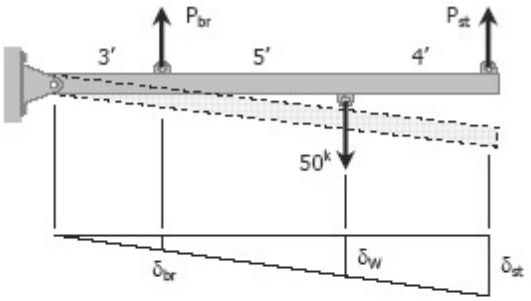


Figure P-253

Solution 253

$$\sum M_{pin\ support} = 0$$
$$3P_{br} + 12P_{st} = 8(50\,000)$$
$$3P_{br} + 12P_{st} = 400\,000 \qquad \rightarrow (1)$$



$$\frac{\delta_{st}}{12} = \frac{\delta_{br}}{3};\ \delta_{st} = 4\delta_{br}$$
$$\left(\frac{PL}{AE}\right)_{st} = 4\left(\frac{PL}{AE}\right)_{br}$$
$$\frac{P_{st}(10)}{0.5(29\times10^6)} = 4\left[\frac{P_{br}(3)}{2(12\times10^6)}\right]$$
$$P_{st} = 0.725P_{br}$$

From equation (1)
$3P_{br} + 12(0.725P_{br}) = 400\,000$
$P_{br} = 34\,188.03$ lb

$$\delta_{br} = \left(\frac{PL}{AE}\right)_{br} = \frac{34188.03(3\times12)}{2(12\times10^6)}$$
$$\delta_{br} = 0.0513 \text{ in}$$

$$\frac{\delta_W}{8} = \frac{\delta_{br}}{3}$$

$$\delta_W = \frac{8}{3}\delta_{br}$$

$$\delta_W = \frac{8}{3}(0.0513)$$

$$\delta_W = \mathbf{0.1368\ in}$$

Check by δ_{st}:

$$P_{st} = 0.725P_{br} = 0.725(34\,188.03)$$

$$P_{st} = 24\,786.32\ \text{lb}$$

$$\delta_{st} = \left(\frac{PL}{AE}\right)_{st}$$

$$\delta_{st} = \frac{24\,786.32(10\times 12)}{0.5(29\times 10^6)}$$

$$\delta_{st} = 0.2051\ \text{in}$$

$$\frac{\delta_W}{8} = \frac{\delta_{st}}{12}$$

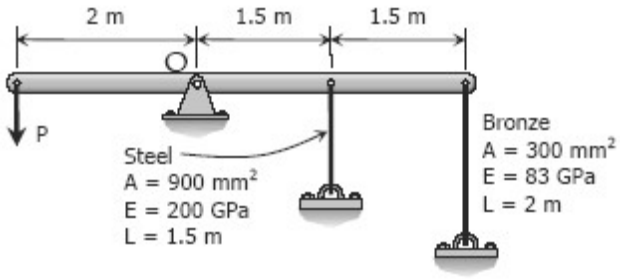
$$\delta_W = \frac{2}{3}\delta_{st}$$

$$\delta_W = \frac{2}{3}(0.2051) = \mathbf{0.1368\ in} \qquad \textit{ok!}$$

Problem 254

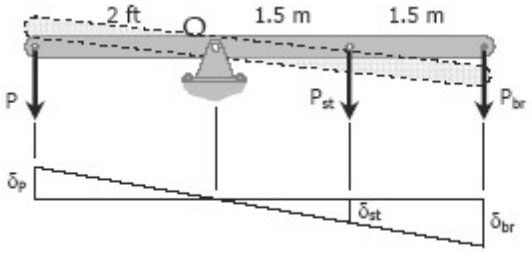
As shown in Fig. P-254, a rigid bar with negligible mass is pinned at O and attached to two vertical rods. Assuming that the rods were initially tress-free, what maximum load P can be applied without exceeding stresses of 150 MPa in the steel rod and 70 MPa in the bronze rod.

Figure P-254



Solution 254

$$\sum M_O = 0$$
$$2P = 1.5P_{st} + 3P_{br}$$
$$2P = 1.5(\sigma_{st}A_{st}) + 3(\sigma_{br}A_{br})$$
$$2P = 1.5[\sigma_{st}(900)] + 3[\sigma_{br}(300)]$$
$$2P = 1350\sigma_{st} + 900\sigma_{br}$$
$$P = 675\sigma_{st} + 450\sigma_{br}$$



$$\frac{\delta_{br}}{3} = \frac{\delta_{st}}{1.5}$$
$$\delta_{br} = 2\delta_{st}$$
$$\left(\frac{\sigma L}{E}\right)_{br} = 2\left(\frac{\sigma L}{E}\right)_{st}$$
$$\frac{\sigma_{br}(2)}{83} = 2\left[\frac{\sigma_{st}(1.5)}{200}\right]$$
$$\sigma_{br} = 0.6225\sigma_{st}$$

When $\sigma_{st} = 150$ MPa
$\sigma_{br} = 0.6225(150)$
$\sigma_{br} = 93.375$ MPa > 70 MPa (*not ok!*)

When $\sigma_{br} = 70$ MPa
$70 = 0.6225\sigma_{st}$
$\sigma_{st} = 112.45$ MPa < 150 MPa (*ok!*)

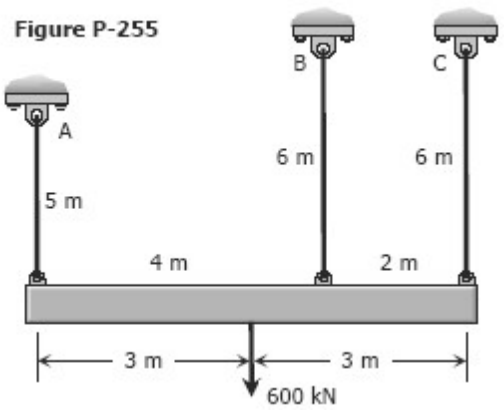
Use $\sigma_{st} = 112.45$ MPa and $\sigma_{br} = 70$ MPa

$$P = 675\sigma_{st} + 450\sigma_{br}$$
$$P = 675(112.45) + 450(70)$$
$$P = 107\,403.75 \text{ N}$$
$$P = \mathbf{107.4\ kN}$$

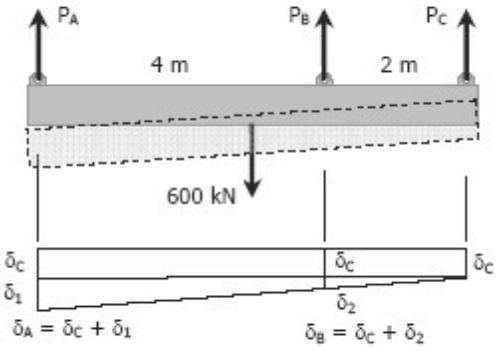
Problem 255

Shown in Fig. P-255 is a section through a balcony. The total uniform load of 600 kN is supported by three rods of the same area and material. Compute the load in each rod. Assume the floor to be rigid, but note that it does not necessarily remain horizontal.

Figure P-255



Solution 255



$$\delta_B = \delta_C + \delta_2$$
$$\delta_2 = \delta_B - \delta_C$$
$$\frac{\delta_1}{6} = \frac{\delta_2}{2};\ \delta_1 = 3\delta_2$$
$$\delta_A = \delta_C + \delta_1 = \delta_C + 3\delta_2$$
$$\delta_A = \delta_C + 3(\delta_B - \delta_C)$$
$$\delta_A = 3\delta_B - 2\delta_C$$
$$\left(\frac{PL}{AE}\right)_A = 3\left(\frac{PL}{AE}\right)_B - 2\left(\frac{PL}{AE}\right)_C$$
$$\frac{P_A(5)}{AE} = \frac{3P_B(6)}{AE} - \frac{2P_C(6)}{AE}$$
$$P_A = 3.6P_B - 2.4P_C \quad \rightarrow (1)$$

$[\sum F_V = 0]$ $P_A + P_B + P_C = 600$

$(3.6P_B - 2.4P_C) + P_B + P_C = 600$

$4.6P_B - 1.4P_C = 600 \quad \rightarrow (2)$

$[\sum M_A = 0]$ $4P_B + 6P_C = 3(600)$

$P_B = 450 - 1.5P_C \quad \rightarrow (3)$

Substitute $P_B = 450 - 1.5P_C$ to (2)

$4.6(450 - 1.5P_C) - 1.4P_C = 600$

$8.3P_C = 1470$

$P_C = \mathbf{177.11\ kN}$

From (3)

$P_B = 450 - 1.5(177.11)$

$P_B = \mathbf{184.34\ kN}$

From (1)

$P_A = 3.6(184.34) - 2.4(177.11)$

$P_A = \mathbf{238.56\ kN}$

Problem 256

Three rods, each of area 250 mm2, jointly support a 7.5 kN load, as shown in Fig. P-256. Assuming that there was no slack or stress in the rods before the load was applied, find the stress in each rod. Use E_{st} = 200 GPa and E_{br} = 83 GPa.

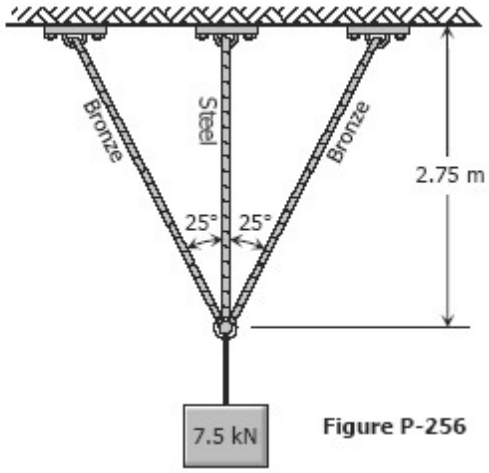
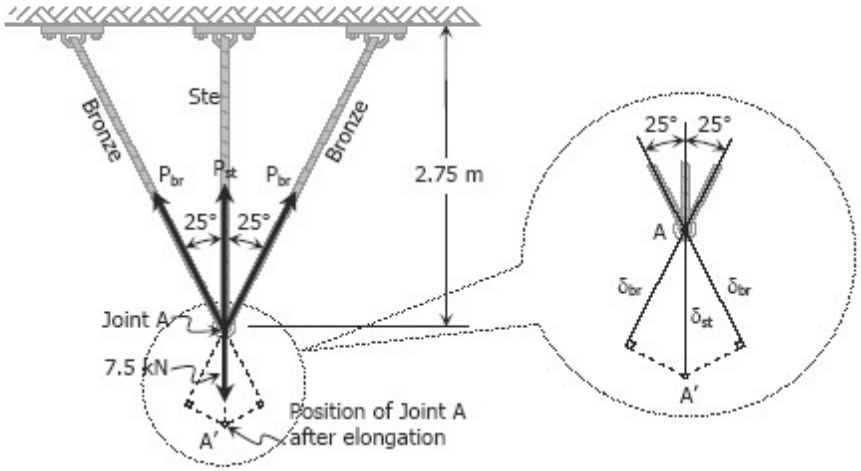


Figure P-256

Solution 256

$$\cos 25° = \frac{2.75}{L_{br}};\ L_{br} = 3.03\ \text{m}$$



$\sum F_V = 0$

$2P_{br} \cos 25° + P_{st} = 7.5(1000)$

$P_{st} = 7500 - 1.8126P_{br}$

$\sigma_{st} A_{st} = 7500 - 1.8126\sigma_{br} A_{br}$

$\sigma_{st} (250) = 7500 - 1.8126[\sigma_{br} (250)]$

$\sigma_{st} = 30 - 1.8126\ \sigma_{br}$ → (1)

$$\cos 25° = \frac{\delta_{br}}{\delta_{st}}$$

$\delta_{br} = 0.9063\ \delta_{st}$

$$\left(\frac{\sigma L}{E}\right)_{br} = 0.9063\left(\frac{\sigma L}{E}\right)_{st}$$

$$\frac{\sigma_{br}(3.03)}{83} = 0.9063\left[\frac{\sigma_{st}(2.75)}{200}\right]$$

$\sigma br = 0.3414\sigma_{st}$ → (2)

From equation (1)

$\sigma_{st} = 30 - 1.8126(0.3414\sigma_{st})$

$\sigma_{st} = \mathbf{18.53\ MPa}$

From equation (2)

$\sigma br = 0.3414(18.53)$

$\sigma_{br} = \mathbf{6.33\ MPa}$

Problem 257

Three bars AB, AC, and AD are pinned together as shown in Fig. P-257. Initially, the assembly is stressfree. Horizontal movement of the joint at A is prevented by a short horizontal strut AE. Calculate the stress in each bar and the force in the strut AE when the assembly is used to support the load W = 10 kips. For each steel bar, A = 0.3 in.^{2} and E = 29 × 10^6 psi. For the aluminum bar, A = 0.6 in.^{2} and E = 10 × 10^6 psi.

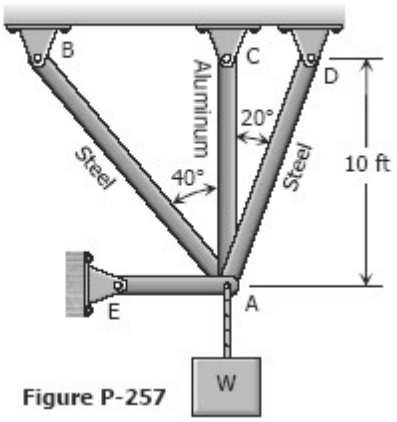
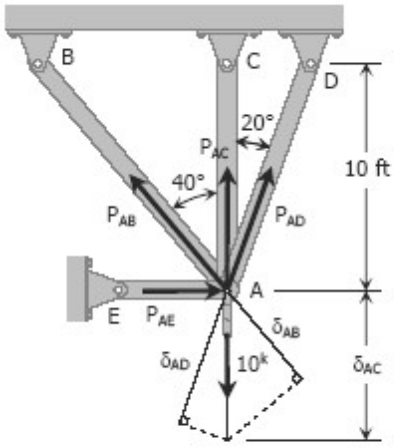


Figure P-257

Solution 257

$\cos 40° = 10 / L_{AB};\ L_{AB} = 13.05$ ft
$\cos 20° = 10 / L_{AD};\ L_{AD} = 10.64$ ft



$\sum F_V = 0$
$P_{AB} \cos 40° + P_{AC} + P_{AD} \cos 20° = 10(1000)$
$0.7660P_{AB} + P_{AC} + 0.9397P_{AD} = 10\,000$ → (1)

$\delta_{AB} = \cos 40°\ \delta_{AC} = 0.7660\ \delta_{AC}$

$$\left(\frac{PL}{AE}\right)_{AB} = 0.7660\left(\frac{PL}{AE}\right)_{AC}$$

$$\frac{P_{AB}(13.05)}{0.3(29\times10^6)} = 0.7660\left[\frac{P_{AC}(10)}{0.6(10\times10^6)}\right]$$

$P_{AB} = 0.8511P_{AC}$ → (2)

$\delta_{AD} = \cos 20°\ \delta_{AC} = 0.9397\ \delta_{AC}$

$$\left(\frac{PL}{AE}\right)_{AD} = 0.9397\left(\frac{PL}{AE}\right)_{AC}$$

$$\frac{P_{AD}(10.64)}{0.3(29\times10^6)} = 0.9397\left[\frac{P_{AC}(10)}{0.6(10\times10^6)}\right]$$

$P_{AD} = 1.2806P_{AC}$ → (3)

Substitute P_{AB} of (2) and P_{AD} of (3) to (1)
$0.7660(0.8511P_{AC}) + P_{AC} + 0.9397(1.2806P_{AC}) = 10\,000$
$2.8553P_{AC} = 10\,000$
$P_{AC} = 3\,502.23$ lb

$P_{AB} = 0.8511(3\,502.23)$ → from (2)
$P_{AB} = 2\,980.75$ lb

$P_{AD} = 1.2806(3\,502.23)$ → from (3)
$P_{AD} = 4\,484.96$ lb

Stresses:
$\sigma = P/A$
$\sigma_{AB} = 2980.75/0.3 =$ **9 935.83 psi**
$\sigma_{AC} = 3502.23/0.6 =$ **5 837.05 psi**
$\sigma_{AD} = 4484.96/0.3 =$ **14 949.87 psi**

$\sum F_H = 0$
$P_{AE} + P_{AD} \sin 20° = P_{AB} \sin 40°$
$P_{AE} = 2\,980.75 \sin 40° - 4\,484.96 \sin 20°$
$P_{AE} =$ **382.04 lb**

Thermal Stress

Temperature changes cause the body to expand or contract. The amount δ_T, is given by

$$\delta_T = \alpha L(T_f - T_i) = \alpha L\, \Delta T$$

where α is the coefficient of thermal expansion in m/m°C, L is the length in meter, and T_i and T_f are the initial and final temperatures, respectively in °C.

For steel, $\alpha = 11.25 \times 10^{-6}$ / °C.

If temperature deformation is permitted to occur freely, no load or stress will be induced in the structure. In some cases where temperature deformation is not permitted, an internal stress is created. The internal stress created is termed as thermal stress.

For a homogeneous rod mounted between unyielding supports as shown, the thermal stress is computed as:

deformation due to temperature changes;

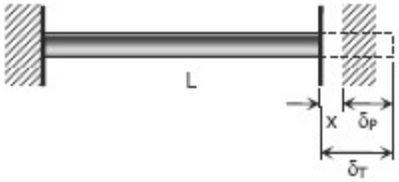
$$\delta_T = \alpha L\, \Delta T$$

deformation due to equivalent axial stress;

$$\delta_P = \frac{PL}{AE} = \frac{\sigma L}{E}$$
$$\delta_T = \delta_P$$
$$\alpha L\, \Delta T = \frac{\sigma L}{E}$$
$$\sigma = E\, \alpha\, \Delta T$$

where σ is the thermal stress in MPa and E is the modulus of elasticity of the rod in MPa.

If the wall yields a distance of x as shown, the following calculations will be made:



$$\delta_T = x + \delta_P$$

$$\alpha L\,\Delta T = x\,\frac{\sigma L}{E}$$

where σ represents the thermal stress.

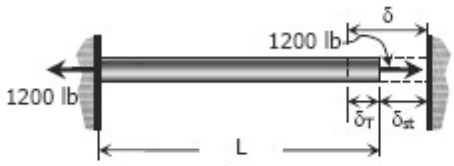
Take note that as the temperature rises above the normal, the rod will be in compression, and if the temperature drops below the normal, the rod is in tension.

Solved Problems in Thermal Stress

Problem 261

A steel rod with a cross-sectional area of 0.25 in^2 is stretched between two fixed points. The tensile load at 70°F is 1200 lb. What will be the stress at 0°F? At what temperature will the stress be zero? Assume $\alpha = 6.5 \times 10^{-6}$ in / (in·°F) and $E = 29 \times 10^6$ psi.

Solution 261



For the stress at 0°C:

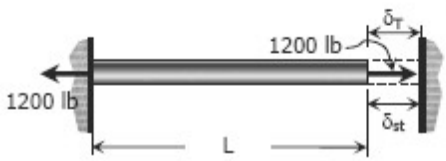
$$\delta = \delta_T + \delta_{st}$$

$$\frac{\sigma L}{E} = \alpha L(\Delta T) + \frac{PL}{AE}$$

$$\sigma = \alpha E(\Delta T) + \frac{P}{A}$$

$$\sigma = (6.5 \times 10^{-6})(29 \times 10^6)(70) + \frac{1200}{0.25}$$

$$\sigma = 17\,995 \text{ psi} = \mathbf{18\ ksi}$$



For the temperature that causes zero stress:

$$\delta_T = \delta_{st}$$

$$\alpha L(\Delta T) = \frac{PL}{AE}$$

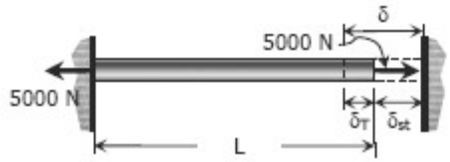
$$(6.5 \times 10^{-6})(T - 70) = \frac{1200}{0.25(29 \times 10^6)}$$

$$T = \mathbf{95.46°C}$$

Problem 262

A steel rod is stretched between two rigid walls and carries a tensile load of 5000 N at 20°C. If the allowable stress is not to exceed 130 MPa at -20°C, what is the minimum diameter of the rod? Assume α = 11.7 μm/(m·°C) and E = 200 GPa.

Solution 262



$$\delta = \delta_T + \delta_{st}$$

$$\frac{\sigma L}{E} = \alpha L(\Delta T) + \frac{PL}{AE}$$

$$\sigma = \alpha E(\Delta T) + \frac{P}{A}$$

$$130 = (11.7 \times 10^{-6})(200\,000)(40) + \frac{5000}{A}$$

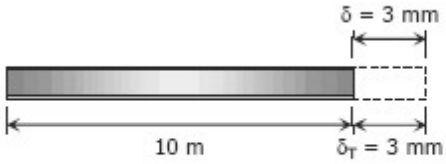
$$A = \frac{5000}{36.4} = 137.36 \text{ mm}^2$$

$$\tfrac{1}{4}\pi d^2 = 137.36; \quad d = \mathbf{13.22 \text{ mm}}$$

Problem 263

Steel railroad reels 10 m long are laid with a clearance of 3 mm at a temperature of 15°C. At what temperature will the rails just touch? What stress would be induced in the rails at that temperature if there were no initial clearance? Assume α = 11.7 μm/(m·°C) and E = 200 GPa.

Solution 263



Temperature at which δ_T = 3 mm:

$$\delta_T = \alpha L(\Delta T)$$

$$\delta_T = \alpha L(T_f - T_i)$$

$$3 = (11.7 \times 10^{-6})(10\,000)(T_f - 15)$$

$$T_f = \mathbf{40.64°C}$$

Required stress:

$$\delta = \delta_T$$

$$\frac{\sigma L}{E} = \alpha L(\Delta T)$$

$$\sigma = \alpha E(T_f - T_i)$$

$$\sigma = (11.7 \times 10^{-6})(200\,000)(40.64 - 15)$$

$$\sigma = \mathbf{60 \text{ MPa}}$$

Problem 264

A steel rod 3 feet long with a cross-sectional area of 0.25 in.^{2} is stretched between two fixed points. The tensile force is 1200 lb at 40°F. Using E = 29 × 10^6 psi and α = 6.5 × 10^{-6} in./(in.·°F), calculate (a) the temperature at which the stress in the bar will be 10 ksi; and (b) the temperature at which the stress will be zero.

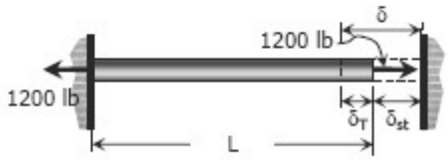
Solution 264

(a) Without temperature change:

$\sigma = P/A = 1200/0.25 = 4800$ psi

$\sigma = 4.8$ ksi < 10 ksi

A drop of temperature is needed to increase the stress to 10 ksi. See accompanying figure.



$$\delta = \delta_T + \delta_{st}$$

$$\frac{\sigma L}{E} = \alpha L(\Delta T) + \frac{PL}{AE}$$

$$\sigma = \alpha E(\Delta T) + \frac{P}{A}$$

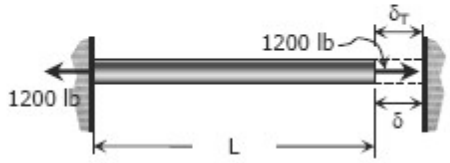
$$10\,000 = (6.5 \times 10^{-6})(29 \times 10^{6})(\Delta T) + \frac{1200}{0.25}$$

$$\Delta T = 27.59°\text{F}$$

Required temperature:
(temperature must drop from 40°F)

$T = 40 - 27.59 = \mathbf{12.41°F}$

(b) From the figure below:



$$\delta = \delta_T$$

$$\frac{PL}{AE} = \alpha L(\Delta T)$$

$$P = \alpha AE(T_f - T_i)$$

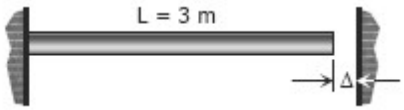
$$1200 = (6.5 \times 10^{-6})(0.25)\,(29 \times 10^{6})(T_f - 40)$$

$$T_f = \mathbf{65.46°F}$$

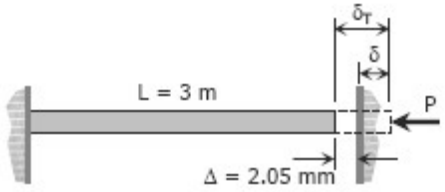
Problem 265

A bronze bar 3 m long with a cross sectional area of 320 mm^2 is placed between two rigid walls as shown in Fig. P-265. At a temperature of -20°C, the gap Δ = 25 mm. Find the temperature at which the compressive stress in the bar will be 35 MPa. Use α = 18.0 × 10^{-6} m/(m·°C) and E = 80 GPa.

Figure P-265



Solution 265



$$\delta_T = \delta + \Delta$$

$$\alpha L(\Delta T) = \frac{\sigma L}{E} + 2.5$$

$$(18 \times 10^{-6})(3000)(\Delta T) = \frac{35(3000)}{80000} + 2.5$$

$$\Delta T = 70.6°\text{C}$$

$$T = 70.6 - 20$$

$$T = \mathbf{50.6°C}$$

Problem 266

Calculate the increase in stress for each segment of the compound bar shown in Fig. P-266 if the temperature increases by 100°F. Assume that the supports are unyielding and that the bar is suitably braced against buckling.

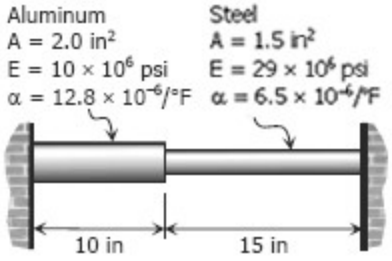
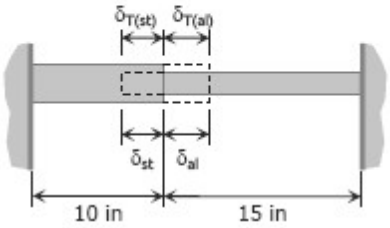


Figure P-266

Solution 266

$$\delta_T = \alpha L \, \Delta T$$



$$\delta_{T(st)} = (6.5 \times 10^{-6})(15)(100)$$
$$\delta_{T(st)} = 0.00975$$

$$\delta_{T(al)} = (12.8 \times 10^{-6})(10)(100)$$
$$\delta_{T(al)} = 0.0128 \text{ in}$$

$$\delta_{st} + \delta_{al} = \delta_{T(st)} + \delta_{T(al)}$$

$$\left(\frac{PL}{AE}\right)_{st} + \left(\frac{PL}{AE}\right)_{al} = 0.00975 + 0.0128$$

where $P = P_{st} = P_{al}$

$$\frac{P(15)}{1.5(29\times10^6)} + \frac{P(10)}{2(10\times10^6)} = 0.02255$$

$$P = 26\,691.84 \text{ psi}$$

$$\sigma = \frac{P}{A}$$

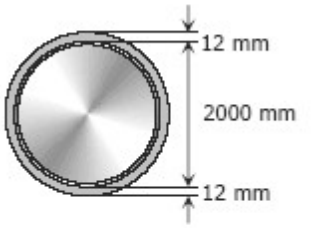
$$\sigma_{st} = \frac{26691.84}{1.5} = \mathbf{17\,794.56 \text{ psi}}$$

$$\sigma_{al} = \frac{26691.84}{2.0} = \mathbf{13\,345.92 \text{ psi}}$$

Problem 267

At a temperature of 80°C, a steel tire 12 mm thick and 90 mm wide that is to be shrunk onto a locomotive driving wheel 2 m in diameter just fits over the wheel, which is at a temperature of 25°C. Determine the contact pressure between the tire and wheel after the assembly cools to 25°C. Neglect the deformation of the wheel caused by the pressure of the tire. Assume α = 11.7 μm/(m·°C) and E = 200 GPa.

Solution 267



$$\delta = \delta_T$$

$$\frac{PL}{AE} = \alpha L \, \Delta T$$

$$P = \alpha \, \Delta T \, AE$$

$$P = (11.7 \times 10^{-6})(80 - 25)(90 \times 12)(200\,000)$$

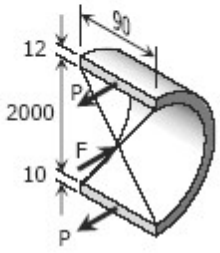
$$P = 138\,996 \text{ N}$$

$$F = 2P$$

$$pDL = 2P$$

$$p(2000)(90) = 2(138996)$$

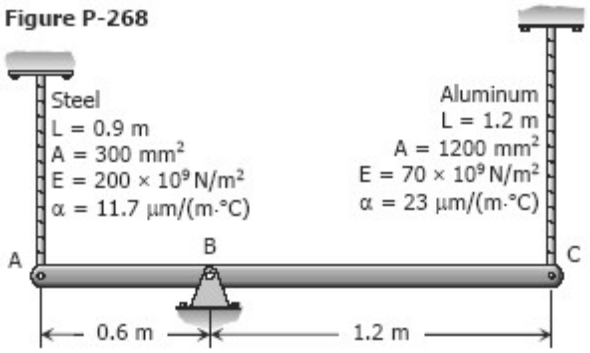
$$p = \mathbf{1.5444 \text{ MPa}}$$



Problem 268

The rigid bar ABC in Fig. P-268 is pinned at B and attached to the two vertical rods. Initially, the bar is horizontal and the vertical rods are stress-free. Determine the stress in the aluminum rod if the temperature of the steel rod is decreased by 40°C. Neglect the weight of bar ABC.

Figure P-268



Solution 268

Contraction of steel rod, assuming complete freedom:

$\delta_{T(st)} = \alpha L \, \Delta T$

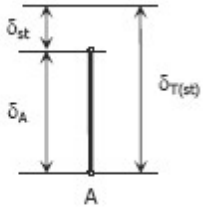
$= (11.7 \times 10^{-6})(900)(40)$

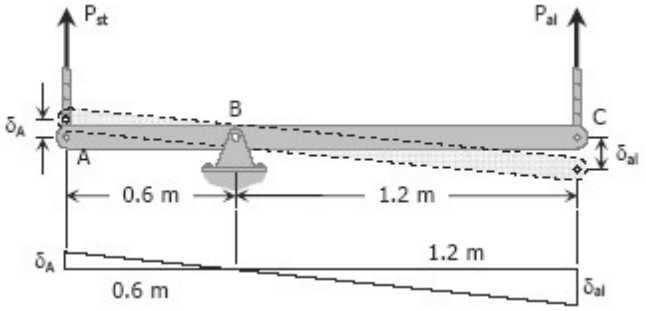
$= 0.4212$ mm

The steel rod cannot freely contract because of the resistance of aluminum rod. The movement of A (referred to as δ_A), therefore, is less than 0.4212 mm. In terms of aluminum, this movement is (by ratio and proportion):

$$\frac{\delta_A}{0.6} = \frac{\delta_{al}}{1.2}$$

$$\delta_A = 0.5\delta_{al}$$





$$\delta_{T(st)} - \delta_{st} = 0.5\,\delta_{al}$$

$$0.4212 - \left(\frac{PL}{AE}\right)_{st} = 0.5\left(\frac{PL}{AE}\right)_{al}$$

$$0.4212 - \frac{P_{st}(900)}{300(200\,000)} = 0.5\left[\frac{P_{al}(1200)}{1200(70\,000)}\right]$$

$28080 - P_{st} = 0.4762P_{al}$ → Equation (1)

$\sum M_B = 0$

$0.6P_{st} = 1.2P_{al}$

$P_{st} = 2P_{al}$ → Equation (2)

Equations (1) and (2)

$28080 - 2P_{al} = 0.4762P_{al}$

$P_{al} = 11340$ N

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{11340}{1200}$$

$\sigma_{al} = \mathbf{9.45\ MPa}$

Problem 269

As shown in Fig. P-269, there is a gap between the aluminum bar and the rigid slab that is supported by two copper bars. At 10°C, Δ = 0.18 mm. Neglecting the mass of the slab, calculate the stress in each rod when the temperature in the assembly is increased to 95°C. For each copper bar, A= 500 mm^2, E = 120 GPa, and α = 16.8 μm/(m·°C). For the aluminum bar, A = 400 mm^2, E = 70 GPa, and α = 23.1 μm/(m·°C).

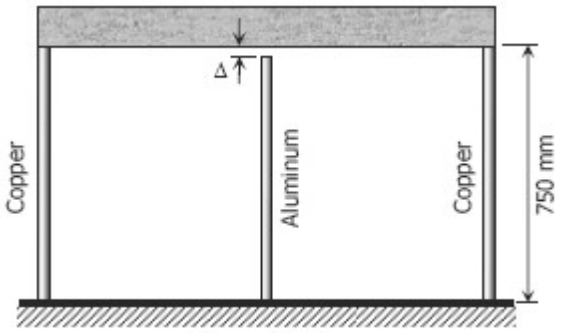
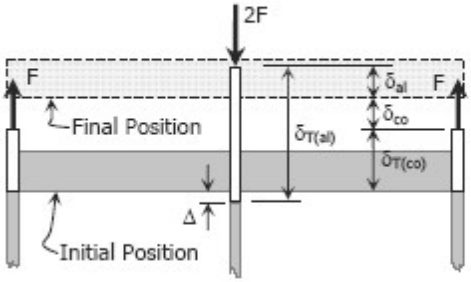


Figure P-269

Solution 269



Assuming complete freedom:

$\delta_T = \alpha L \, \Delta T$

$\delta_{T(co)} = (16.8 \times 10^{-6})(750)(95 - 10)$

$= 1.071$ mm

$\delta_{T(al)} = (23.1 \times 10^{-6})(750 - 0.18)(95 - 10)$

$= 1.472$ mm

From the figure:

$\delta_{T(al)} - \delta_{al} = \delta_{T(co)} + \delta_{co}$

$$1.472 - \left(\frac{PL}{AE}\right)_{al} = 1.071 + \left(\frac{PL}{AE}\right)_{co}$$

$$1.472 - \frac{2F(750 - 0.18)}{400(70\,000)} = 1.071 + \frac{F(750)}{500(120\,000)}$$

$0.401 = (6.606 \times 10^{-5})\, F$

$F = 6070.37$ N

$P_{co} = F = 6070.37$ N

$P_{al} = 2F = 12\,140.74$ N

$\sigma = P/A$

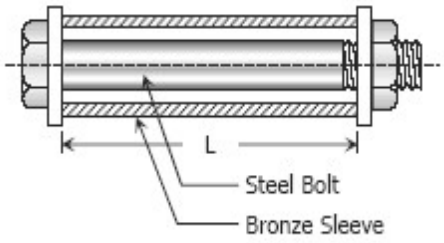
$$\sigma_{co} = \frac{6070.37}{500} = \mathbf{12.14\ MPa}$$

$$\sigma_{al} = \frac{12\,140.74}{400} = \mathbf{30.35\ MPa}$$

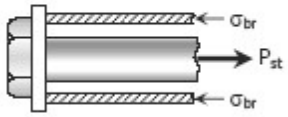
Problem 270

A bronze sleeve is slipped over a steel bolt and held in place by a nut that is turned to produce an initial stress of 2000 psi in the bronze. For the steel bolt, A = 0.75 in^2, E = 29 × 10^6 psi, and α = 6.5 × 10^{-6} in/(in·°F). For the bronze sleeve, A = 1.5 in^2, E = 12 × 10^6 psi and α = 10.5 × 10^{-6} in/(in·°F). After a temperature rise of 100°F, find the final stress in each material.

Solution 270



Before temperature change:



$P_{br} = \sigma_{br} A_{br}$

$= 2000(1.5)$

$= 3000$ lb compression

$\Sigma F_H = 0$

$P_{st} = P_{br} = 3000$ lb tension

$\sigma_{st} = P_{st}/A_{st} = 3000/0.75$

$= 4000$ psi tensile stress

$$\delta = \frac{\sigma L}{E}$$

$$a = \delta_{br} = \frac{2000L}{12\times10^6} = 1.67 \times 10^{-4}L \text{ shortening}$$

$$b = \delta_{st} = \frac{4000L}{29\times10^6} = 1.38 \times 10^{-4}L \text{ lengthening}$$

With temperature rise of 100°F:

(Assuming complete freedom)

$\delta_T = \alpha L\, \Delta T$

$\delta_{Tbr} = (10.5 \times 10^{-6})L\ (100)$

$= 1.05 \times 10^{-3}L > a$

$\delta_{Tst} = (6.5 \times 10^{-6})L\ (100)$

$= 6.5 \times 10^{-4}L$

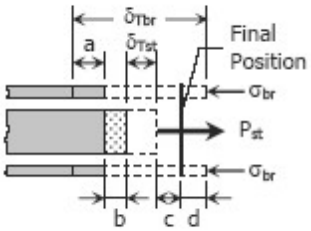
$\delta_{Tbr} - a = 1.05 \times 10^{-3}L - 1.67 \times 10^{-4}L$

$= 8.83 \times 10^{-4}L$

$\delta_{Tst} + b = 6.5 \times 10^{-4}L + 1.38 \times 10^{-4}L$

$= 7.88 \times 10^{-4}L$

$\delta_{Tbr} - a > \delta_{Tst} + b$ (see figure below)



$\delta_{Tbr} - a - d = b + \delta_{Tst} + c$

$$1.05 \times 10^{-3}L - 1.67 \times 10^{-4}L - \left(\frac{\sigma L}{E}\right)_{br}$$

$$= 1.38 \times 10^{-4}L + 6.5 \times 10^{-4}L + \left(\frac{PL}{AE}\right)_{st}$$

$$8.83 \times 10^{-4}L - \frac{\sigma_{br} L}{12\times10^6}$$

$$= 7.88 \times 10^{-4}L + \frac{P_{st} L}{0.75(29\times10^6)}$$

$$9.5 \times 10^{-4} - \frac{P_{br}}{1.5(12 \times 10^6)} = \frac{P_{st}}{0.75(29 \times 10^6)}$$

$P_{st} = 20\,662.5 - 1.2083P_{br}$ → Equation (1)

$\Sigma F_H = 0$

$P_{br} = P_{st}$ → Equation (2)

Equations (1) and (2)

$P_{st} = 20\,662.5 - 1.2083P_{st}$

$P_{st} = 9356.74$ lb

$P_{br} = 9356.74$ lb

$\sigma = P/A$

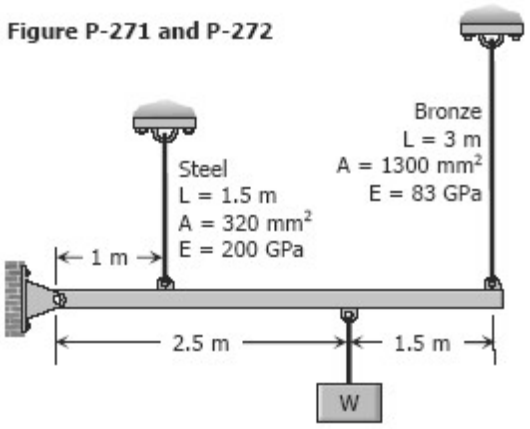
$\sigma_{br} = \dfrac{9356.74}{1.5} = 6237.83$ psi compressive stress

$\sigma_{st} = \dfrac{9356.74}{0.74} = 12\,475.66$ psi tensile stress

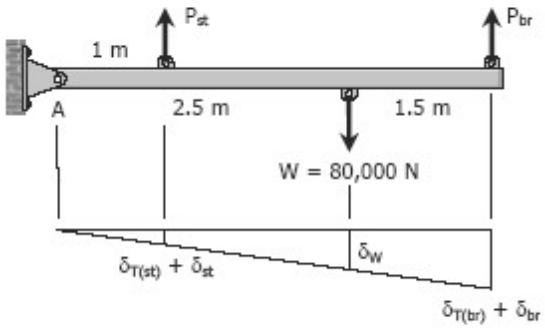
Problem 271

A rigid bar of negligible weight is supported as shown in Fig. P-271. If W = 80 kN, compute the temperature change that will cause the stress in the steel rod to be 55 MPa. Assume the coefficients of linear expansion are 11.7 μm/(m·°C) for steel and 18.9 μm / (m·°C) for bronze.

Figure P-271 and P-272



Solution 271



Stress in bronze when $\sigma_{st} = 55$ MPa

$$\Sigma M_A = 0$$
$$4P_{br} + P_{st} = 2.5(80000)$$
$$4\sigma_{br}(1300) + 55(320) = 2.5(80000)$$
$$\sigma_{br} = 35.08 \text{ MPa}$$

By ratio and proportion:

$$\frac{\delta_{T(st)} + \delta_{st}}{1} = \frac{\delta_{T(br)} + \delta_{br}}{4}$$
$$\delta_{T(st)} + \delta_{st} = 0.25[\delta_{T(br)} + \delta_{br}]$$
$$(\alpha L\,\Delta T)_{st} + \left(\frac{\sigma L}{E}\right)_{st} = 0.25\left[(\alpha L\,\Delta T)_{br} + \left(\frac{\sigma L}{E}\right)_{br}\right]$$
$$(11.7 \times 10^{-6})(1500)\,\Delta T + \frac{55(1500)}{2000} = 0.25\left[(18.9 \times 10^{-6})(3000)\,\Delta T + \frac{35.08(3000)}{83000}\right]$$
$$0.017\,55\,\Delta T + 0.4125 = 0.014\,175\,\Delta T + 0.317$$
$$\Delta T = -28.3\ ^\circ\text{C}$$

A temperature drop of 28.3 °C is needed to stress the steel to 55 MPa.

Problem 272

For the assembly in Fig. 271, find the stress in each rod if the temperature rises 30°C after a load W = 120 kN is applied.

Solution 272

$$\Sigma M_A = 0$$
$$4P_{br} + P_{st} = 2.5(80000)$$
$$4\sigma_{br}(1300) + \sigma_{st}(320) = 2.5(80000)$$
$$16.25\sigma_{br} + \sigma_{st} = 625$$
$$\sigma_{st} = 625 - 16.25\sigma_{br} \quad \rightarrow \text{Equation (1)}$$

$$\frac{\delta_{T(st)} + \delta_{st}}{1} = \frac{\delta_{T(br)} + \delta_{br}}{4}$$
$$\delta_{T(st)} + \delta_{st} = 0.25[\delta_{T(br)} + \delta_{br}]$$

$$(\alpha L\,\Delta T)_{st} + \left(\frac{\sigma L}{E}\right)_{st} = 0.25\left[(\alpha L\,\Delta T)_{br} + \left(\frac{\sigma L}{E}\right)_{br}\right]$$

$$(11.7 \times 10^{-6})(1500)(30) + \frac{\sigma_{st}(1500)}{200000}$$

$$= 0.25\left[(18.9 \times 10^{-6})(3000)(30) + \frac{\sigma_{br}(3000)}{83000}\right]$$

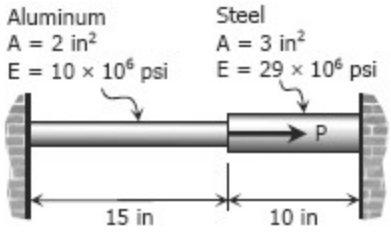
$0.5265 + 0.0075\sigma_{st} = 0.425\,25 + 0.00904\sigma_{br}$
$0.0075\sigma_{st} - 0.00904\sigma_{br} = -0.10125$
$0.0075(625 - 16.25\sigma_{br}) - 0.00904\sigma_{br} = -0.10125$
$4.6875 - 0.121\,875\sigma_{br} - 0.009\,04\sigma_{br} = -0.101\,25$
$4.788\,75 = 0.130\,915\sigma_{br}$
$\sigma_{br} = 36.58$ °C

$\sigma_{st} = 625 - 16.25(36.58)$
$\sigma_{st} = 30.58$ °C

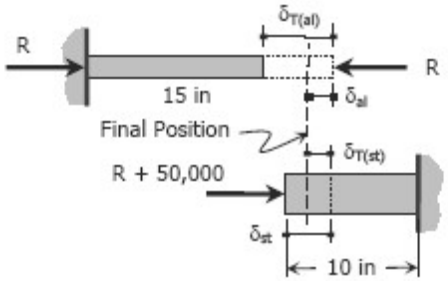
Problem 273

The composite bar shown in Fig. P-273 is firmly attached to unyielding supports. An axial force P = 50 kips is applied at 60°F. Compute the stress in each material at 120°F. Assume $\alpha = 6.5 \times 10^{-6}$ in/(in·°F) for steel and 12.8×10^{-6} in/(in·°F) for aluminum.

Figure P-273 and P-274



Solution 273



$\delta_{T(al)} = (\alpha L\,\Delta T)_{al}$
$\delta_{T(al)} = (12.8 \times 10^{-6})(15)(120 - 60)$
$\delta_{T(al)} = 0.011\,52$ inch

$\delta_{T(st)} = (\alpha L\,\Delta T)_{st}$
$\delta_{T(st)} = (6.5 \times 10^{-6})(10)(120 - 60)$
$\delta_{T(st)} = 0.0039$ inch

$$\delta_{T(al)} - \delta_{al} = \delta_{st} - \delta_{T(st)}$$

$$0.011\,52 - \left(\frac{PL}{AE}\right)_{al} = \left(\frac{PL}{AE}\right)_{st} - 0.0039$$

$$0.011\,52 - \frac{R(15)}{2(10\times10^6)} = \frac{(R+50\,000)(10)}{3(29\times10^6)} - 0.0039$$

$$100\,224 - 6.525R = R + 50\,000 - 33\,930$$

$$84\,154 = 7.525R$$

$$R = 11\,183.25 \text{ lbs}$$

$$P_{al} = R = 11\,183.25 \text{ lbs}$$

$$P_{st} = R + 50\,000 = 61\,183.25 \text{ lbs}$$

$$\sigma = \frac{P}{A}$$

$$\sigma_{al} = \frac{11\,183.25}{2}$$

$$= \mathbf{5\,591.62 \text{ psi}}$$

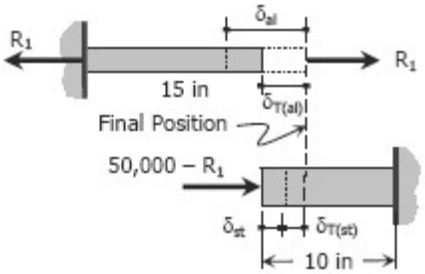
$$\sigma_{st} = \frac{61\,183.25}{3}$$

$$= \mathbf{20\,394.42 \text{ psi}}$$

Problem 274

At what temperature will the aluminum and steel segments in Prob. 273 have numerically equal stress?

Solution 274



$$\sigma_{al} = \sigma_{st}$$

$$\frac{R_1}{2} = \frac{(50\,000 - R_1)}{3}$$

$$3R_1 = 100\,000 - 2R_1$$

$$R_1 = 20\,000 \text{ lbs}$$

$$\delta = \frac{PL}{AE}$$

$$\delta_{al} = \frac{20\,000(15)}{2(10\times10^6)} = 0.015 \text{ inch}$$

$$\delta_{st} = \frac{(50\,000 - 20\,000)(10)}{3(29\times10^6)} = 0.003\,45 \text{ inch}$$

$$\delta_{al} - \delta_{T(al)} = \delta_{st} + \delta_{T(st)}$$

$$0.015 - (12.8\times10^{-6})(15)\,\Delta T = 0.003\,45 + (6.5\times10^{-6})(10)\,\Delta T$$

$$0.011\,55 = 0.000\,257\,\Delta T$$

$$\Delta T = 44.94°\text{F}$$

A drop of **44.94°F** from the standard temperature will make the aluminum and steel segments equal in stress.

Problem 275

A rigid horizontal bar of negligible mass is connected to two rods as shown in Fig. P-275. If the system is initially stress-free. Calculate the temperature change that will cause a tensile stress of 90 MPa in the brass rod. Assume that both rods are subjected to the change in temperature.

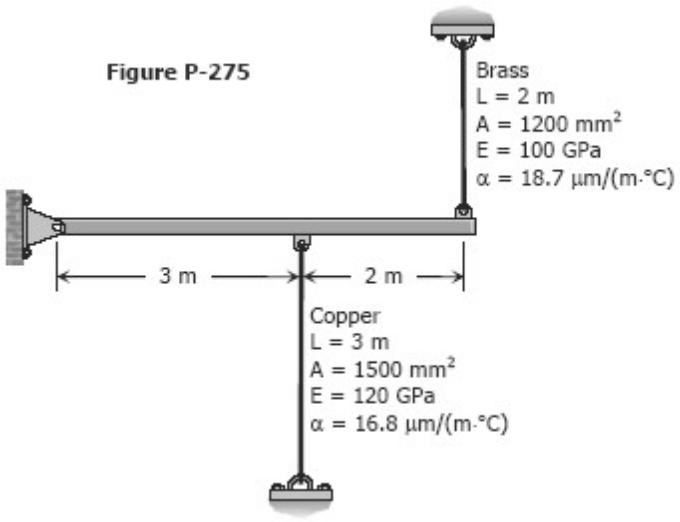
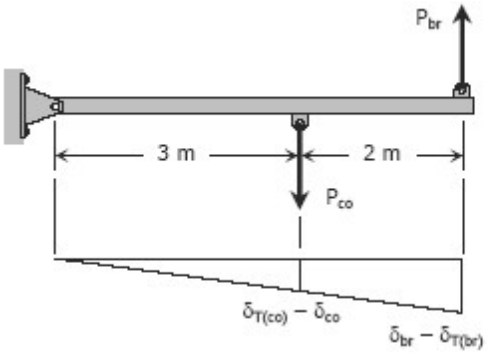


Figure P-275

Solution 275



$\Sigma M_{\text{hinge support}} = 0$

$5P_{br} - 3P_{co} = 0$

$5\sigma_{br} A_{br} - 3\sigma_{co} A_{co} = 0$

$5(90)(1200) - 3\sigma_{co}(1500) = 0$

$\sigma_{co} = 120 \text{ MPa}$

$\delta = \sigma L / E$

$\delta_{br} = 90(2000) / 100\,000 = 1.8 \text{ mm}$

$\delta_{co} = 120(3000) / 120\,000 = 3 \text{ mm}$

$$\frac{\delta_{T(co)} - \delta_{co}}{3} = \frac{\delta_{br} - \delta_{T(br)}}{5}$$

$5\delta_{T(co)} - 5\delta_{co} = 3\delta_{br} - 3\delta_{T(br)}$

$5(16.8\times10^{-6})(3000)\,\Delta T - 5(3)$

$= 3(1.8) - 3(18.7\times10^{-6})(2000)\,\Delta T$

$0.3642\,\Delta T = 20.4$

$\Delta T = 56.01°\text{C}$ **drop in temperature**

Problem 276

Four steel bars jointly support a mass of 15 Mg as shown in Fig. P-276. Each bar has a cross-sectional area of 600 mm2. Find the load carried by each bar after a temperature rise of 50°C. Assume α = 11.7 μm/(m·°C) and E = 200 GPa.

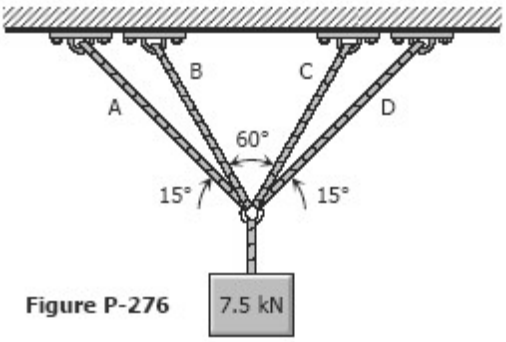
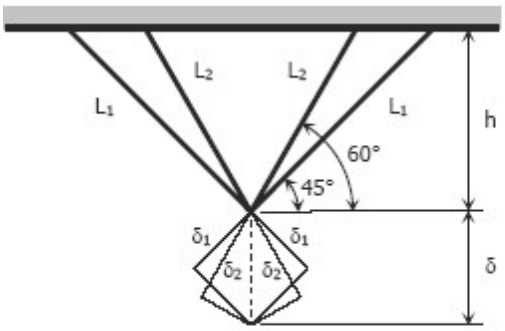


Figure P-276

Solution 276



$h = L_1 \sin 45°$
$h = L_2 \sin 60°$

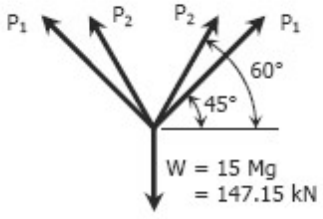
$h = h$
$L_1 \sin 45° = L_2 \sin 60°$
$L_1 = 1.2247L_2$

$\delta_1 = \delta \sin 45°$
$\delta_2 = \delta \sin 60°$
$\dfrac{\delta_1}{\delta_2} = \dfrac{\delta \sin 45°}{\delta \sin 60°}$
$\delta_1 = 0.8165\delta_2$

$$\alpha L_1 \Delta T + \frac{P_1 L_1}{AE} = 0.8165\left[\alpha L_2 \Delta T + \frac{P_2 L_2}{AE}\right]$$

$$(11.7 \times 10^{-6})L_1 (50) + \frac{P_1 L_1}{600(200000)} = 0.8165\left[(11.7 \times 10^{-6})L_2 (50) + \frac{P_2 L_2}{600(200000)}\right]$$

$70{,}200L_1 + P_1L_1 = 0.8165(70{,}200L_2 + P_2L_2)$
$(70{,}200 + P_1)L_1 = 0.8165(70{,}200 + P_2)L_2$
$(70{,}200 + P_1)1.2247\cancel{L_2} = 0.8165(70{,}200 + P_2)\cancel{L_2}$
$1.5(70{,}200 + P_1) = 70{,}200 + P_2$
$P_2 = 1.5P_1 + 35{,}100$ → Equation (1)



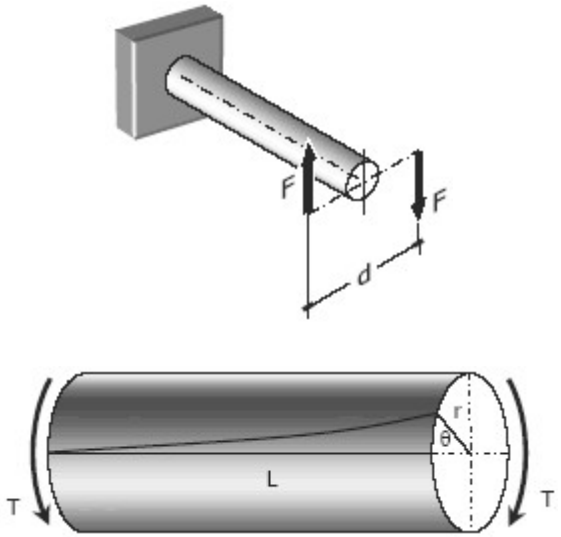
$\Sigma F_V = 0$
$2(P_1 \sin 45°) + 2(P_2 \sin 60°) = 147.15(1000)$
$P_1 \sin 45° + P_2 \sin 60° = 72{,}575$
$P_1 \sin 45° + (1.5P_1 + 35{,}100) \sin 60° = 72{,}575$
$0.7071P_1 + 1.299P_1 + 30{,}397.49 = 72{,}575$
$2.0061P_1 = 42{,}177.51$
$P_1 = 21{,}024.63$ N

$P_2 = 1.5(21{,}024.63) + 35{,}100$
$P_2 = 66{,}636.94$ N

$P_A = P_D = P_1 = \mathbf{21.02\ kN}$
$P_B = P_C = P_2 = \mathbf{66.64\ kN}$

Torsion

Consider a bar to be rigidly attached at one end and twisted at the other end by a torque or twisting moment T equivalent to F × d, which is applied perpendicular to the axis of the bar, as shown in the figure. Such a bar is said to be in torsion.



TORSIONAL SHEARING STRESS, τ

For a solid or hollow circular shaft subject to a twisting moment T, the torsional shearing stress τ at a distance ρ from the center of the shaft is

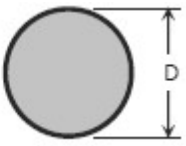
$$\tau = \frac{T\rho}{J} \text{ and } \tau_{max} = \frac{Tr}{J}$$

where J is the polar moment of inertia of the section and r is the outer radius.

For solid cylindrical shaft:

$$J = \frac{\pi}{32}D^4$$

$$\tau_{max} = \frac{16T}{\pi D^3}$$



For hollow cylindrical shaft:

$$J = \frac{\pi}{32}(D^4 - d^4)$$

$$\tau_{max} = \frac{16TD}{\pi(D^4 - d^4)}$$